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参数曲面上的插值与混合*

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Interpolation and Blending on Parametric Surfaces

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Abstract: Representing a curve contained in a surface is very important in dealing with path generation in computer numerical control (CNC) machining and the trimming issues that frequently occur in the field of CAD/CAM. This paper develops methods for tangent direction continuous (G^1) and both tangent direction and curvature continuous (G^2) interpolation of a range of points on surface with specified tangent and either a curvature vector or a geodesic curvature at every point. As a special case of the interpolation, the blending problems of curves on surface are also discussed**.** The basic idea is as follows: with the help of the related results of differential geometry, the problem of interpolating curve on a parametric surface is converted to a similar one on its parametric plane. The methods can express the G^1 and G^2 interpolation curve of an arbitrary sequence of points on a parametric surface in a 2D implicit form, which transforms the geometric problem of surface intersection, usually a troublesome issue, into the algebraic problem of computing an implicit curve in displaying such an interpolation curve. Experimental results show the presented methods are feasible and applicable to CAD/CAM and Computer Graphics.

Key words: interpolation; blending; G^1 continuity; G^2 continuity; tangent mapping

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WANG Xiao-Ping was born in 1966. He is a postdoctoral research fellow in Research Center of CAD/CAM Engineering, Nanjing University of Aeronautics and Astronautics. His research interests are Geometric Modeling and Computer Graphics. **ZHOU Ru-Rong** was born in 1936. He is a professor and doctoral supervisor. He has been devoted to the research in the field of CAD/CAPP/CAM/CAE/ PDE, and to the development and generalization of applications for over 20 years. **YU Zhan-Yue** was born in 1973. He is a Ph.D. candidate majoring in CAD/CAM. **YE Zheng-Lin** was born in 1946. He is a professor and doctoral supervisor. His research areas cover Computational Geometry, Computer Graphics and Image Processing.

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摘 要: 如何表示曲面上的曲线,在处理诸如数控加工中的路径设计以及 CAD/CAM 等领域频繁出现的曲面裁剪问题时显得日益重要.给出了数据点的切方向(切方向及曲率向量或测地曲率值)指定而 G^1 连续(G^2 连续)插值曲面上任意点列的方法.作为曲面上曲线插值问题的特例,还讨论了曲面上曲线的混合问题.基本思想是借助于微分几何的有关结论,曲面上曲线的插值问题被转化为其参数平面上类似的曲线插值问题.该方法能够用二维隐式方程来表示曲面上的插值曲线,从而把在显示该曲线时所面对的曲面求交的几何问题转化为计算隐式曲线的代数问题.实验证明该方法是可行的,而且适用于 CAD/CAM 及计算机图形学等领域.
关键词: 插值;混合;G^1 连续;G^2 连续;切映射
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In the fields of CAD/CAM, computer graphics, computer animation, robotics, CNC machining and so on, numerous problems involve the representation of a space or a planar curve. There is an extensive range of literatures touching upon the problems. However, so far, only several efforts have been made towards developing more effective methods for representation of surface curves (curves contained in the specified surfaces). Pobegailo proposed an approach for G^1 interpolation and blending on a sphere[1]. Dietz et al. solved G^0 interpolation problem on quadrics for the prescribed pairs $(P_1,t_1),\dots,(P_n,t_n)$ of points and parameters with the help of rational Bézier curves [2]. Hartmann developed a method for curvature-continuous (G^2) interpolation of an arbitrary sequence of points on a surface (implicit or parametric) with the specified tangent and a geodesic curvature at every point [3], which can be directly employed in G^2 blending of curves on surfaces. Other related researches focus on G^1 and C^1 interpolation presented in Refs.[4~5] respectively. Apparently, the method in Ref.[5] is good for display of the resulting interpolation curve. However the resulting interpolation curve is a composed curve that might have too high degree. In addition, direct approximation such as that with piecewise 4-point Bézier cubic curves or linear curves was also applied to the representation of surface curves in practical applications[6,7]. In fact, almost all trim-related literatures introduce approximate methods. The aim of this paper is to develop methods for G^1 and G^2 interpolation of an arbitrary sequence of points on surfaces with a prescribed tangent and curvature at any points. With the help of relevant results in differential geometry, we convert the problems of space G^1 and G^2 interpolation on surfaces into the similar ones in parametric plane. Other contributions in this paper include that we obtain the corresponding curve in parametric plane of a space G^1 and G^2 interpolation curve on a regular surface and discuss the blending issues of surface curves (implicit or parametric) such that the blending curve segment can be described in an implicit form.

The rest of the paper is organized in the following manner. Section 1 introduces the necessary mathematical bases that are involved in the presented methods for the representation of surface curves, while problem statements are given in Section 2. Interpolation and blending issues are discussed in Section 3, where two kinds of methods to represent surface curve are developed in Sections 3.1 and 3.2 respectively, and the blending method is developed in Subsection 3.3. Practical examples and comparisons are given in Section 4. Finally, Section 5 finishes the paper with conclusions.

1 Mathematical Preliminary

Suppose $r(u,v)=(x(u,v),y(u,v),z(u,v))^T$, $u,v\in[0,1]$ is a C^{r_1} regular surface[8]. Furthermore taking the equation of a C^{r_2} curve in (u,v) plane as $u=u(t)$, $v=v(t)$, $t\in[a,b]$ and substituting them into the above surface equation, we get a space $C^{\min[r_1,r_2]}$ curve:

$$\boldsymbol{r}(t)=(x(u(t),v(t)),\ y(u(t),v(t)),z(u(t),v(t)))=(x^*(t),\ y^*(t),\ z^*(t))^T,\quad t\in[a,b]$$

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which is contained in the surface and called the *surface curve*. Obviously the curve $u=u(t), v=v(t)$ is the *original image curve* of the curve $\boldsymbol{r}(t)$ under the mapping $\boldsymbol{r}$: $[0,1]\times[0,1]\to R^3$.

Now let us consider the relations between tangent vectors, between their second derived vectors, and between their curvatures of the space curve $\boldsymbol{r}(t)$ and its original curve in parametric plane respectively. Write the original image curve as $\boldsymbol{\alpha}(t)=(u(t),v(t))^T$. From the derivation formula of composite function, i.e., chain rule, it follows that:

$$\boldsymbol{r}'=\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t}=\boldsymbol{r}_u u'+\boldsymbol{r}_v v'=(\boldsymbol{r}_u\ \ \boldsymbol{r}_v)\begin{pmatrix}u'\\ v'\end{pmatrix}=\begin{pmatrix}\partial x/\partial u & \partial x/\partial v\\ \partial y/\partial u & \partial y/\partial v\\ \partial z/\partial u & \partial z/\partial v\end{pmatrix}\begin{pmatrix}\mathrm{d}u/\mathrm{d}t\\ \mathrm{d}v/\mathrm{d}t\end{pmatrix}, \tag{1}$$

$$\begin{aligned}\boldsymbol{r}''&=\frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}t^2}=\boldsymbol{r}_{uu}(u')^2+2\boldsymbol{r}_{uv}u'v'+\boldsymbol{r}_{vv}(v')^2+\boldsymbol{r}_u u''+\boldsymbol{r}_v v''\\
&=\left((\boldsymbol{r}_{uu}\ \ \boldsymbol{r}_{uv})\begin{pmatrix}u'\\ v'\end{pmatrix}\ \ (\boldsymbol{r}_{uv}\ \ \boldsymbol{r}_{vv})\begin{pmatrix}u'\\ v'\end{pmatrix}\right)\begin{pmatrix}u'\\ v'\end{pmatrix}+(\boldsymbol{r}_u\ \ \boldsymbol{r}_v)\begin{pmatrix}u''\\ v''\end{pmatrix}\\
&=\left(\begin{pmatrix}\partial^2 x/\partial u^2 & \partial^2 x/\partial u\partial v\\ \partial^2 y/\partial u^2 & \partial^2 y/\partial u\partial v\\ \partial^2 z/\partial u^2 & \partial^2 z/\partial u\partial v\end{pmatrix}\begin{pmatrix}\mathrm{d}u/\mathrm{d}t\\ \mathrm{d}v/\mathrm{d}t\end{pmatrix}\ \ \begin{pmatrix}\partial^2 x/\partial u\partial v & \partial^2 x/\partial v^2\\ \partial^2 y/\partial u\partial v & \partial^2 y/\partial v^2\\ \partial^2 z/\partial u\partial v & \partial^2 z/\partial v^2\end{pmatrix}\begin{pmatrix}\mathrm{d}u/\mathrm{d}t\\ \mathrm{d}v/\mathrm{d}t\end{pmatrix}\right)\begin{pmatrix}\mathrm{d}u/\mathrm{d}t\\ \mathrm{d}v/\mathrm{d}t\end{pmatrix}+\\
&\quad\begin{pmatrix}\partial x/\partial u & \partial x/\partial v\\ \partial y/\partial u & \partial y/\partial v\\ \partial z/\partial u & \partial z/\partial v\end{pmatrix}\begin{pmatrix}\mathrm{d}^2u/\mathrm{d}t^2\\ \mathrm{d}^2v/\mathrm{d}t^2\end{pmatrix}.\end{aligned} \tag{2}$$

Assume $\boldsymbol{k}$ and k_g denote the curvature vector and geodesic curvature of curve $\boldsymbol{r}(t)$ respectively, and k_α is the curvature of its original image curve $\boldsymbol{\alpha}(t)$ at the corresponding point. From differential geometry of curve, we have

$$\begin{aligned}\frac{\boldsymbol{r}'\times\boldsymbol{k}}{|\boldsymbol{r}'|}=\frac{\boldsymbol{r}'\times\boldsymbol{r}''}{|\boldsymbol{r}'|^3}&=\frac{((\boldsymbol{r}_u\ \boldsymbol{r}_v)\boldsymbol{\alpha}')\times(((\boldsymbol{r}_{uu}\ \boldsymbol{r}_{uv})\boldsymbol{\alpha}'\,(\boldsymbol{r}_{uv}\ \boldsymbol{r}_{vv})\boldsymbol{\alpha}')\boldsymbol{\alpha}')+\boldsymbol{r}_u\times\boldsymbol{r}_v(v''u'-u''v')}{|(\boldsymbol{r}_u\ \boldsymbol{r}_v)\boldsymbol{\alpha}'|^3}\\
&=\frac{((\boldsymbol{r}_u\ \boldsymbol{r}_v)\boldsymbol{\alpha}')\times(((\boldsymbol{r}_{uu}\ \boldsymbol{r}_{uv})\boldsymbol{\alpha}'\,(\boldsymbol{r}_{uv}\ \boldsymbol{r}_{vv})\boldsymbol{\alpha}')\boldsymbol{\alpha}')+\boldsymbol{r}_u\times\boldsymbol{r}_v k_\alpha\,|\boldsymbol{\alpha}'|^3}{|(\boldsymbol{r}_u\ \boldsymbol{r}_v)\boldsymbol{\alpha}'|^3}\end{aligned} \tag{3}$$

Obviously, it is difficult for us to get an explicit expression satisfied by the curvature $|\boldsymbol{k}|$ and k_α of the surface curve. However, let's turn to k_g for help. Suppose N is the unit normal vector of surface at a specified point, then from (3), we get

$$k_g=\frac{[((\boldsymbol{r}_u\ \boldsymbol{r}_v)\boldsymbol{\alpha}')\times(((\boldsymbol{r}_{uu}\ \boldsymbol{r}_{uv})\boldsymbol{\alpha}'\,(\boldsymbol{r}_{uv}\ \boldsymbol{r}_{vv})\boldsymbol{\alpha}')\boldsymbol{\alpha}')]\cdot\boldsymbol{N}+k_\alpha\,|\boldsymbol{r}_u\times\boldsymbol{r}_v|\cdot|\boldsymbol{\alpha}'|^3}{|(\boldsymbol{r}_u\ \boldsymbol{r}_v)\boldsymbol{\alpha}'|^3} \tag{4}$$

Then from (3) and noting the fact $\boldsymbol{r}'\perp\boldsymbol{k}$, the following result can easily be got:

Proposition 1. At a given point with a specified tangent direction, the curvature vector of a surface curve on a regular surface and the curvature of its original image curve determine each other uniquely.

From (4), it follows that:

Proposition 2. At a given point with a specified tangent direct, the geodesic curvature of a surface curve on a regular surface and the curvature of its original image curve determine each other uniquely,

By Propositions 1 and 2, we further conclude:

Proposition 3. At the corresponding point, a surface curve on a regular surface and its original image curve have the same continuity such as that of position, tangent vector and curvature.

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Proposition 4. A surface curve on a regular surface is G^1 continuous and curvature–continuous (vector), if and only if its original image curve is G^1 continuous and curvature–continuous at the corresponding point, and the similar conclusions hold for geodesic curvature.

Now let us consider the surface. In fact, it is defined by a mapping $\boldsymbol{r}$: $[0,1]\times[0,1]\rightarrow R^3$. Since we assume the surface is a regular surface, the mapping is an one-to-one mapping and the *tangent mapping* induced by it is an isomorphic mapping between tangent spaces of the plane domain and that of the surface at the respective corresponding points. Using $T_r(\boldsymbol{r}(t_0))$ and $T_\alpha(\boldsymbol{\alpha}(t_0))$ to denote the tangent spaces of surface at the point $\boldsymbol{r}(t_0)$ and corresponding plane domain at the point $\boldsymbol{\alpha}(t_0)$ respectively, then the tangent mapping $d\boldsymbol{r}: T_\alpha(\boldsymbol{\alpha}(t_0))\rightarrow T_r(\boldsymbol{r}(t_0))$ is a linear one-to-one mapping (isomorphic). By differential geometry[8], this mapping can be expressed in the following matrix form:

$$\mathrm{d}\boldsymbol{r}(\boldsymbol{X})=\begin{pmatrix}\partial x/\partial u & \partial x/\partial v\\ \partial y/\partial u & \partial y/\partial v\\ \partial z/\partial u & \partial z/\partial v\end{pmatrix}\boldsymbol{X}=\begin{pmatrix}\boldsymbol{r}_u & \boldsymbol{r}_v\end{pmatrix}\boldsymbol{X} \tag{5}$$

where the column vector $\boldsymbol{X}\in T_\alpha(\boldsymbol{\alpha}(t_0))$. For a regular surface, the transform matrix of (5) satisfies $rank(\boldsymbol{r}_u \quad \boldsymbol{r}_v)=2$. So from (1) and (2), we get the following conclusion:

Proposition 5. At corresponding points, the tangent vector of surface curve on a regular surface and that of its original image curve determine each other uniquely, and the similar case is true to their second derived vectors.

As for a concrete computation, please refer to formulae (1) and (2).

2 Problem Statements

Since most CAD systems adopt parametric representations for free-form shapes, we mainly consider the issue of interpolation curve on parametric surfaces.

Problem 1. Given an arbitrary sequence $\boldsymbol{r}_i$, i=1,…,s, of points on a C^r regular surface, where $r\geq 2$, find an interpolation curve passing them with tangent direction $\boldsymbol{t}_i$ at corresponding point $\boldsymbol{r}_i$.

Problem 2. Find an interpolation curve passing an arbitrary sequence $\boldsymbol{r}_i$, i=1,…,s, of points on a C^r regular surface ($r\geq 3$) on conditions that the curve's tangent direction and geodesic curvatures or curvature vectors at any points $\boldsymbol{r}_i$ are specified.

Problem 3. Given two curves on a surface, find a transition curve on the surface that connects the known curves at two specified points with G^1 or G^2 continuity at the two points.

3 Interpolation and Blending on Surfaces

In this section, we want to solve the above problems with the so-called *functional spline* method[9]. Hartmann et al. once used this method successfully in creating a G^2 interpolation curve expressed by the intersection curve of a given surface and a functional spline surface (implicit)[3]. However, with the presented method, the interpolation is processing in parametric plane other than in space. In addition, the last interpolation curve is represented implicitly by a plane curve in parametric plane instead of by the intersection of two surfaces which always involves complicated algorithms of finding surface- to-surface intersection when there is a need to display the interpolation curve, for example, when people handle the trimming surface problem.

3.1 Interpolation problem 1

First, we target problem 1. It is sufficient to consider the interpolation curve defined by only a pair of points, such as $\boldsymbol{r}_1,\boldsymbol{r}_2$ with the prescribed unit vectors $\boldsymbol{T}_1,\boldsymbol{T}_2$ at the corresponding points on the surface $\boldsymbol{r}(u,v)$ since our

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tactics is a piecewise interpolation. As points on the regular surface, $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ correspond uniquely to the *original image points* respectively, say $\boldsymbol{\alpha}_1=(u_1,v_1)$ and $\boldsymbol{\alpha}_2=(u_2,v_2)$ in parametric plane under the mapping $\boldsymbol{r}$. We indeed are able to solve the original image points through traditional Newton iteration method. However we strongly recommend another method presented in Ref.[10] that has a better compute stability and a quicker convergence rate than the traditional methods. Sometimes if we want to get an exact image point instead of an approximate one, then such methods as the resultant method[11], the Gröbner Base method[12] and Sederberg method[13] can all be used to deal with the issue. Moreover, from Proposition 5, under the tangent mapping $\mathrm{d}\boldsymbol{r}$, the tangent vectors $\boldsymbol{T}_1,\boldsymbol{T}_2$ at corresponding points $\boldsymbol{r}_1,\boldsymbol{r}_2$ of the desired interpolation curve determine respectively their original image vectors that belong to the tangent spaces at their corresponding points in parametric plane. Let us assume the original image vectors are $\boldsymbol{t}_1,\boldsymbol{t}_2$ respectively, which actually are the tangent vectors at points $\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2$ of the plane curve determined uniquely by the desired interpolation curve on surface. In fact, they can be computed by Eq.(1) or (5). See Figs.1~2.

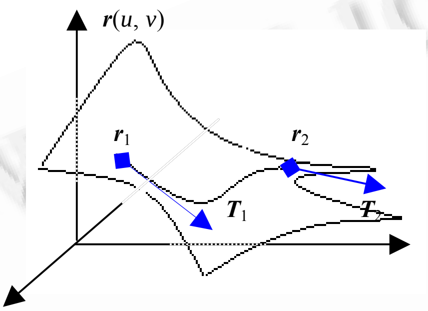

Fig.1 Interpolation on surface

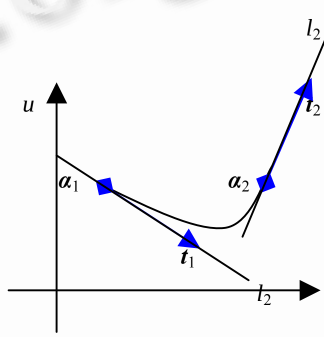

Fig.2 Interpolation on parametric plane

Now the problem of interpolation on surface is reduced to the same problem on plane, which is easy to solve referring to the results[3,9]. Before getting the equation of interpolation curve, the following assumptions are necessary. Let $\boldsymbol{x}_1=(u_1,v_1),\boldsymbol{x}_2=(u_2,v_2)$ be the coordinate vectors of the points $\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2$ in parametric plane respectively, $g_i(\boldsymbol{x})=\boldsymbol{n}_i(\boldsymbol{x}-\boldsymbol{x}_i)=0$ the normal equation of straight line l_i which passes $\boldsymbol{\alpha}_i$ along the vector $\boldsymbol{t}_i$, $i=1,2$, and $g_{12}(\boldsymbol{x})=\boldsymbol{n}_{12}(\boldsymbol{x}-\boldsymbol{x}_1)=0$ the connecting line equation of the points $\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2$, where $\boldsymbol{x}=(u,v)$. In addition, the assumptions $g_1(\boldsymbol{x}_2)>0$, $g_2(\boldsymbol{x}_1)>0$ (otherwise, we multiply g_1 or g_2 by "−1") are also necessary. Then from Li *et al*.[9] the desired interpolation curve that possesses the tangent vectors $\boldsymbol{t}_1,\boldsymbol{t}_2$ at the points $\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2$ respectively can be expressed as follows:

$$(1-\mu)g_1g_2+\mu g_{12}^2=0,\quad 0<\mu<1 \tag{6}$$

The constant μ can be used as a shape parameter in adjusting the shape of the interpolation curve. Furthermore let $\boldsymbol{\alpha}_i$ be the original image point of $\boldsymbol{r}_i$ in parametric plane, $\boldsymbol{T}_i$ the unit tangent vector of the desired curve at the point $\boldsymbol{r}_i$, and $\boldsymbol{t}_i$ the unit original image vector of $\boldsymbol{T}_i$ under the tangent mapping $\mathrm{d}\boldsymbol{r}$. If $g_i(\boldsymbol{x})=0$, $i=1,\dots,s$ are the normal equations of straight line l_i that passes $\boldsymbol{\alpha}_i$ along the vector $\boldsymbol{t}_i$, $i=1,\dots,s$, then the solution of problem 1 can be expressed piecewisely as follows:

$$\boldsymbol{r}(u,v)=(x(u,v),y(u,v),z(u,v))^T \text{ and } (1-\mu_i)g_ig_{i+1}+\mu_i g_{i,i+1}^2=0,\quad 0<\mu_i<1 \tag{7}$$

where $g_{i,\,i+1}(\boldsymbol{x})=\boldsymbol{n}_{i,\,i+1}(\boldsymbol{x}-\boldsymbol{x}_i)=0$ is the normal equation of the straight line connecting $\boldsymbol{\alpha}_i$ and $\boldsymbol{\alpha}_{i+1}$, $i=1,\dots,s-1$.

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Similarly all μ_i , $i=1,\ldots,s-1$, are shape parameters for the last shape modification of the whole interpolation curve.

Remark 1. Referring to Hartmann's method[3], we can easily get a G^2 continuous surface curve with the curvature vector at one of those specified interpolation points. Here all μ_i , $i=1,\ldots,s-1$, are fixed values determined by the prescribed curvature.

3.2 Interpolation problem 2

As for problem 2, we only consider the case that curvature vector of the interpolation points are specified (the cases for a given geodesic curvature can be handled similarly). Given a sequence of triplets $(\boldsymbol{r}_i,\boldsymbol{T}_i,\boldsymbol{k}_i)$, i=1,…,s, where $\boldsymbol{T}_i$ and $\boldsymbol{k}_i$ are the tangent vector and the curvature of the desired curve at point $\boldsymbol{r}_i$ respectively, from Propositions 1 and 5 we know every triplet $(\boldsymbol{r}_i,\boldsymbol{T}_i,\boldsymbol{k}_i)$ has an unique original image triplet corresponding to itself under the mapping $\boldsymbol{r}$ and its tangent mapping $d\boldsymbol{r}$. Writing it as $(\boldsymbol{a}_i,\boldsymbol{t}_i,k_i)$, where $\boldsymbol{t}_i$ and k_i are the tangent vector and the curvature of the corresponding curve in parametric plane at point $\boldsymbol{a}_i$ respectively, they can be computed by (1) or (5) and (3) respectively with the given $\boldsymbol{T}_i$ and $\boldsymbol{k}_i$. Now the problem is reduced to interpolating the triplet $(\boldsymbol{a}_i,\boldsymbol{t}_i,k_i)$, i=1,…,s, in plane. Similar to dealing with problem 1, we only consider one pair of triplet, i.e., the case s=2. Let us assume that $t_{1,2}$ is an unit vector with the direction $\boldsymbol{a}_2-\boldsymbol{a}_1$, $\boldsymbol{t}_1$ is not parallel to $\boldsymbol{t}_{1,2}$, and $\boldsymbol{x}_1=(u_1,v_1),\boldsymbol{x}_2=(u_2,v_2)$ represent the coordinate vectors of the points $\boldsymbol{a}_1$, $\boldsymbol{a}_2$ respectively, then the implicit equation of the circle that passes the point $\boldsymbol{a}_1$ and possesses the tangent vector $\boldsymbol{t}_1$ and the curvature k_1 at $\boldsymbol{a}_1$ is

$$f_1(\boldsymbol{x})=\left(\boldsymbol{x}-\boldsymbol{x}_1-\frac{1}{k_1}\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\boldsymbol{t}_1\right)^2-\left(\frac{1}{k_1}\right)^2=0 \tag{8}$$

Similarly, we can get another circle that passes the point $\boldsymbol{a}_2$ and possesses the tangent vector $\boldsymbol{t}_2$ and the curvature k_2 at $\boldsymbol{a}_2$. Its equation is

$$f_2(\boldsymbol{x})=\left(\boldsymbol{x}-\boldsymbol{x}_2-\frac{1}{k_2}\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\boldsymbol{t}_2\right)^2-\left(\frac{1}{k_2}\right)^2=0 \tag{9}$$

Next, take the straight line determined by the two points $\boldsymbol{a}_1,\boldsymbol{a}_2$ as the transversal curve that is needed in constructing a desired function spline. Let the equation of the straight line be $g_{12}(\boldsymbol{x})=\boldsymbol{n}_{12}(\boldsymbol{x}-\boldsymbol{x}_1)=0$. Assume $f_1(\boldsymbol{x}_2)>0$ and $f_2(\boldsymbol{x}_1)>0$ (otherwise, we multiply f_1 or f_2 by "–1"). Then from Li *et al*.[9] the equation of curve that interpolates the triplets $(\boldsymbol{a}_i,\boldsymbol{t}_i,k_i)$, i=1,2, in parametric plane is

$$f(\boldsymbol{x})=(1-\mu)f_1f_2-\mu g_{12}^3=0,\quad 0<\mu<1.$$

Its corresponding surface curve that interpolate the triplets $(\boldsymbol{r}_i,\boldsymbol{T}_i,k_{n,i})$, $i=1,2$, on the given surface is

$$\boldsymbol{r}(u,v)=(x(u,v),y(u,v),z(u,v))^T \text{ and } f(\boldsymbol{x})=(1-\mu)f_1f_2-\mu g_{12}^3=0,\quad 0<\mu<1 \tag{10}$$

Thus, analogous to problem 1, it is easy to get the solution to problem 2.

Remark 2. Interpolation curve (7) or (10) possesses local property, i.e., changing one point or one tangent vector or one curvature vector affects the shape of two neighboring curve segments while changing one shape parameter μ_i affects only the shape of one corresponding curve segment.

Remark 3. As for displaying the surface curve (7) or (10), we need such an algorithm for tracing an implicit plane curve as that described in Refs.[14~16].

Remark 4. As a by-production, this kinds of representation methods for surface curves such as that in solutions to problems 1 and 2 also solves the problem of representation of the bound curve in parametric plane that

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determines the deformation regions[17].

3.3 Blending curves on surfaces——problem 3

Actually, the problem of *blending curve* (as defined in Ref.[3]) on surfaces is a special case of interpolation curves on surfaces. So those interpolation methods described in the above sections can be used directly in constructing a G^1 or G^2 blending curve (transition curve) between two given curves parametrically or implicitly on a parametric surface. The blending curve is completely determined by the tangents or tangents and curvatures at the two ends of the transition curve segment and has nothing to do with the global geometry and representation of the two given curves. In contrast to the general interpolation problem, we must first specify two points on two curves, compute the tangent directions or curvatures of two surface curves at the two points respectively, and use them as interpolation conditions. Then the remaining work for us to do is similar to dealing with the interpolation issue. As for curvature computation of surface curves with all kinds of expression forms, one can consult the formulae given out in Ref.[3].

4 Examples and Comparisons

For the sake of simplicity, we take a paraboloid for example and construct an interpolation curve on it to demonstrate the presented method. Let its equation be:

$$\boldsymbol{r}(u,v)=(u,v,(-u^2-v^2)/8+9/4),\quad (u,v)\in[-4,4]\times[-4,4].$$

Specify the interpolation conditions on the paraboloid as follows:

$$\boldsymbol{r}_1=(-2\sqrt{2},0,5/4),\ \boldsymbol{T}_1=(5,5,5\sqrt{2}/2);\ \boldsymbol{r}_2=(2,-2,5/4),\ \boldsymbol{T}_2=(2,-4,3).$$

According to Section 3.1 we obtain the corresponding interpolation conditions on parametric plane:

$$\boldsymbol{\alpha}_1=(-2\sqrt{2},0),\ \boldsymbol{t}_1=(5,5);\ \boldsymbol{\alpha}_2=(2,-2),\ \boldsymbol{t}_2=(2,-4).$$

Construct a planar interpolation curve, which passes the two points $\boldsymbol{\alpha}_1$, $\boldsymbol{\alpha}_2$ with the corresponding tangent vectors $\boldsymbol{t}_1$, $\boldsymbol{t}_2$. See Fig.4. Here we take the shape parameter (see Section 3.1) as $\mu=0.17$, then the equation of the surface curve (see Fig.3) is

$$\boldsymbol{r}(u,v)=(u,v,(-u^2-v^2)/8+9/4),\quad f(\boldsymbol{x})=f(u,v)=0.$$

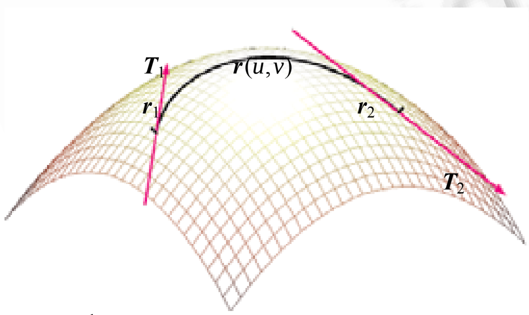

Fig.3 G^1 interpolation curve segment on a paraboloid

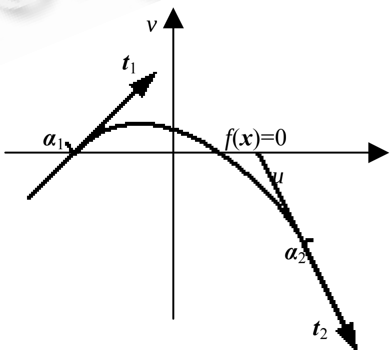

Fig.4 Interpolation curve segment on parametric plane

Still taking a paraboloid as an example, we construct a surface interpolation curve passing the given end points with the specified tangent directions and curvature vector at the end points.

Let's adopt the interpolation conditions on surface as follows:

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$$\boldsymbol{r}_1=(-2\sqrt{2},0,5/4),\ \boldsymbol{T}_1=(0,-2\sqrt{2},0),\ \boldsymbol{k}_1=(\sqrt{2}/4,0,0);\ \boldsymbol{r}_2=(2,-2,5/4),\ \boldsymbol{T}_2=(2,2,0),\ \boldsymbol{k}_2=(-1/4,1/4,0),$$

where $\boldsymbol{k}_1$, $\boldsymbol{k}_2$ denote the curvature vectors of the desired surface curve at the two end points. Compute their counterparts in parametric plane and write them as follows:

$$\boldsymbol{\alpha}_1=(-2\sqrt{2},0),\ \boldsymbol{t}_1=(0,-2\sqrt{2}),\ k_{\alpha 1}=\sqrt{2}/4;\ \boldsymbol{\alpha}_2=(2,-2,),\ \boldsymbol{t}_2=(2,2),\ k_{\alpha 2}=\sqrt{2}/4.$$

Then using the method described in Section 3.2, we get the equation of the original image curve $f(\boldsymbol{x})=0$. Finally, analogous to (10), the equation of the desired surface curve can be obtained. See Figs.5~6, where the shape parameter is taken as $\mu=0.35$.

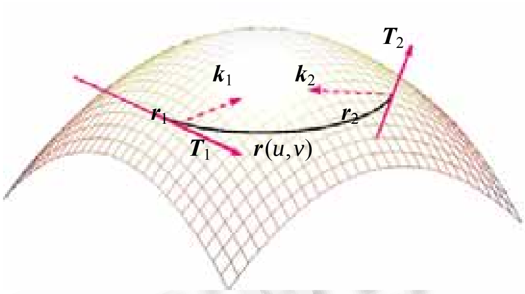

Fig.5 G^2 Interpolation curve segment on a paraboloid

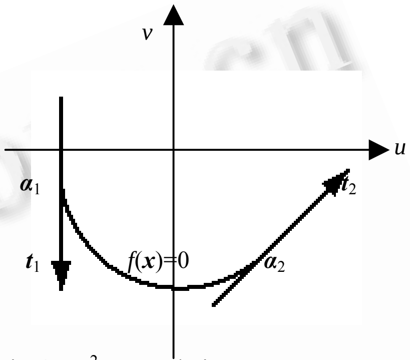

Fig 6 G^2 Interpolation curve segment on parametric plane

Now we construct a G^1 continuous interpolation curve on the paraboloid. Take the interpolation conditions on the surface as follows:

$$\boldsymbol{r}_{11}=(-2\sqrt{2},0,5/4),\ \boldsymbol{T}_{11}=(0,-2\sqrt{2},0);\ \boldsymbol{r}_{12}=(2,-2,5/4),\ \boldsymbol{T}_{12}=(2,2,0);$$
$$\boldsymbol{r}_{21}=(2,-2,5/4),\ \boldsymbol{T}_{21}=(5,5,0);\ \boldsymbol{r}_{22}=(0,0,9/4),\ \boldsymbol{T}_{22}=(-2,-2,0).$$

Analogously, compute their counterparts on parametric plane and write them as follows:

$$\boldsymbol{\alpha}_{11}=(-2\sqrt{2},0),\ \boldsymbol{t}_{11}=(0,-2\sqrt{2});\ \boldsymbol{\alpha}_{12}=(2,-2),\ \boldsymbol{t}_{12}=(2,2);$$
$$\boldsymbol{\alpha}_{21}=(2,-2),\ \boldsymbol{t}_{21}=(5,5);\ \boldsymbol{\alpha}_{22}=(0,0),\ \boldsymbol{t}_{22}=(-2,-2).$$

Then construct planar interpolation curve $f(\boldsymbol{x})=0$ (see Fig.8). The desired curve is the image curve of the curve $f(\boldsymbol{x})=0$ under the mapping $\boldsymbol{r}(u,v)$. See Fig.7.

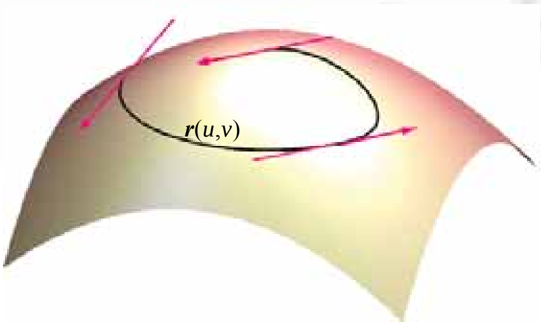

Fig.7 G^2 continuous interpolation curve on a paraboloid

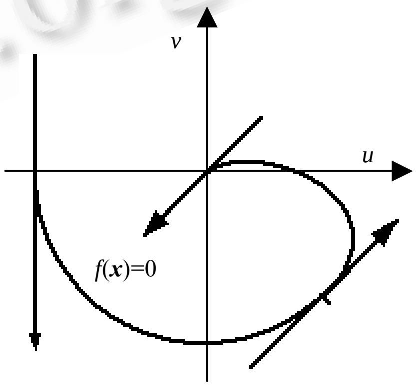

Fig.8 The original image curve on parametric plane

The examples presented above demonstrate the method is effective. Compared with the existing main method[3], its interpolation process proceeds in parametric plane rather than in space. It can be used in the cases that

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not only the geodesic curvature but also curvature vector at every interpolation point is prescribed (in Figs.5~6, curvature vector instead of geodesic curvature are prescribed as interpolation data). Moreover, it involves only tracing an implicit planar curve instead of any surface-to-surface algorithm, usually a troublesome process, on which the method [3] often depends, for displaying the resulting interpolation curves. In addition, Ref.[4] also reports a G^1 interpolation method. In contrast to this method, the interpolation curve generated by the presented method obviously has good controllability since we introduce free shape parameters into every interpolation curve segment that can be used in interactive modification, and interpolation process does not need to perform any curve-to-surface intersection. Reference [5] describes a C^1 interpolation, of which the resulting interpolation curve is a composed curve. Unfortunately, a curve generated by composition might have a very high degree; For a bi-cubic surface composed with a cubic curve, the resulting surface curve has a degree 18 that might be prohibited in most CAD systems. Compared with it, this method relaxes the limitation of interpolation conditions, avoids high degree of interpolation curve, and has more comprehensive applicability in CAD engineering practice.

5 Conclusions

Approaches for the representation of surface curves have been developed. The main idea of the methods and their marked difference from the existing methods lie in the fact that we transform the problem of representation of the surface curves into the one of representation of the planar curves and that the distribution of interpolation points can be arbitrary. The concrete steps of the method are summed up as follows:

- Prescribe the interpolation information such as points, tangent vectors and curvature on surface.
- Compute the corresponding interpolation information on parametric plane.
- Construct planar interpolation curve.

Though we only pay an attention to the representation of curves contained in parametric surface in Section 4.1, in fact, the method can deal with the representation of curves contained in such an implicit surface that can be parameterized. What is more, with the help of the thoughts of the presented method, many good methods designed for general interpolation curves can be used in dealing with the issue of interpolation curves on surfaces.

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第 13 届全国信息存储技术学术会议

征文通知

为促进和加强存储技术的学术交流和产品展示，中国计算机学会信息存储技术专业委员会决定于 2004 年 10 月中旬在西安召开第 13 届全国信息存储技术学术会议。本次会议由中国计算机学会信息存储技术专业委员会主办，西北工业大学计算机学院承办。会议将通过学术报告、专题讨论、产品展示等多种形式，就信息存储的最新研究进展和发展趋势开展深入、广泛的学术交流，并特邀著名专家学者作专题报告。

一、 征文范围

欢迎从事信息技术研究、开发、应用的各界人士，就下列领域(但不限于)所涉及的信息存储技术方面的内容踊跃来稿：国内外存储技术的发展现状及趋势、信息存储理论研究与信息存储新技术研究；计算机主存体系结构研究及实现、海量信息存储技术、网络存储技术；存储领域中的核心技术及实现研究、存储相关芯片的设计与应用、智能存储技术；多媒体信息存储技术、数据仓库、数据挖掘；信息存储系统的安全性和可靠性;存储系统解决方案、存储技术及产品的标准。

二、 征文要求

应征学术论文应是未正式发表过的研究成果，字数(含中英文摘要、关键字与参考文献)不超过 8000 字。请注明作者的通信地址、邮政编码、电话和 E-mail 地址。论文格式请参照《计算机研究与发展》格式。

投稿方式：电子投稿。稿件请采用 Word、PDF 文档格式。

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征文截止: 2004 年 07 月 15 日　录用通知: 2004 年 08 月 31 日

被本次会议录用的学术论文将收录在会议论文集内，优秀论文将在《计算机研究与发展》（增刊）上发表。有关会议的动态信息可通过拨打(029)8493772 或通过 E-mail 询问。

三、参展范围

为了加强产业界、学术界和应用领域间的交流和联系，本次会议将举办信息存储技术交流会及产品展示会。凡与存储相关的产品和技术均欢迎参会参展，有意参展的单位请联系：

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