1000-9825/2002/13(04)0482-08 ©2002 Journal of Software 软 件 学 报 Vol.13, No.4

3D Geometric Constraint Solving with Conicoid*

JIANG Kun[1,2], GAO Xiao-shan[1]

[1](*Academy of Mathematics and System Sciences, The Chinese Academy of Sciences, Beijing* 100080, *China*);

[2](*College of Science, Heilongjiang University, Harbin* 150080, *China*)

E-mail: {kjiang,xgao}@mmrc.iss.ac.cn

http://www.mmrc.iss.ac.cn

Received March 1, 2001; accepted September 14, 2001

Abstract: In general, most 3D parametric design systems use plane and sphere as basic tools to draw a three-dimensional diagram. In this paper, we introduce a class of new drawing tools: conicoid. The scope of three-dimensional diagram that can be drawn with conicoid is strictly larger than that with plane and sphere only. We prove that a diagram can be drawn with conicoid sequentially if and only if it can be described by a set of triangular equations of degree less than nine. Spatial Appolonius drawing problems can be completely solved with conicoid.

Key words: geometric constraint solving; conicoid; parametric CAD

Contemporary CAD (computer aided design) systems have been proven to be efficient for generating engineering drawings and modeling 3D objects. However, in the conception stage of the design process, most CAD systems still do not have all of the required flexibility. Designers are required to know beforehand the precise dimensions of the geometries, and changes are difficult. Nevertheless, in the product development cycle, several design changes are typically needed before the full requirements for functionality, manufacture and quality are met. Therefore, traditional CAD systems are not suitable for most conceptual design. These facts imply the need for a flexible tool for the conceptual design. A potential tool to meet these requirements is parametric design systems which provide the designers a way to create the initial design without knowing exact dimensions[1~3]. Moreover parametric design allows designers to make modifications to existing initial designs by changing parameter values. Parametric design has been incorporated into various CAD/CAM systems, such as Pro/Engineer and I-DEAS Master Series. This kind of parametric modeling systems have promised to revolutionize mechanical CAD/CAM systems.

Geometry Constraint Solving (GCS) is the central topic in much of the current work of developing intelligent or parametric CAD system[4~9]. Geometric constraint solving also applied to solid modeling, graphics, engineering, computer vision, and many other fields. There are three major approaches to GCS: numerical approach, symbolic approach and constructive approach. In the constructive approach, a pre-treatment is carried out to transform the constraint problem into a constructive form[1,3,5,9,10] that is easy to draw. Once a constraint diagram is transformed into constructive form, all its solutions can be computed efficiently. Hence most parametric design systems adopt

* Supported by the National Grand Fundamental Research 973 Program of China under Grant No.G1998030600 (国家重点基础研究发展规划973项目基金); the Chinese National Science Fundation under an Outstanding Youth Grant No.69725002 (国家基金委杰出青年基金)

JIANG Kun was born in 1972. He is a postdoctor at the Academy of Mathematics and System Sciences, the Chinese Academy of Sciences. His current research areas include intelligent CAD and CAI. **GAO Xiao-shan** was born in 1963. He is a professor and doctoral supervisor of the Academy of Mathematics and System Sciences, the Chinese Academy of Sciences. His current research areas are automated deduction, intelligent CAD and computer algebra.

the constructive approach as a basic scheme for GCS. Other approaches are used if the constructive approach fails to give a solution.

Three-dimensional geometric constraint solving is a rapidly developing field. So far there exits little literature in this field. In Ref.[11] a graph-based algorithm is given for three-dimensional geometric constraint solving that only considers points and plane as basic geometric objects. In Refs.[12,13], a systematic framework is presented for solving algebraic equations arising in geometric constraint solving. The framework has been used successfully to solve a family of spatial geometric constraint problems. The approach combines geometric reasoning, symbolic reduction, and homotopy continuation. In this paper we will extend the rule-based approach proposed in Ref.[10] to solve 3D geometric constraint problems. In general, most 3D parametric design systems use plane and sphere as basic tools to draw a three-dimensional diagram. In this paper, we introduce a class of new drawing tools: conicoid. The scope of three-dimensional diagram that can be drawn with conicoid is strictly larger than that with plane and sphere only. We prove that a diagram can be drawn with conicoid sequentially if and only if it can be described by a set of triangular equations of degree less than nine.

1 3D geometric Constraint Solving with Conicoid

The geometric constraint problem considered in this paper is 3D, that is, geometric objects and geometric constraints upon the geometric objects are 3D. Geometric objects could be points, planes and spheres. Geometric constraints could be distance, angle, perpendicular, parallel, tangent, etc.

1.1 Constructive sequence with conicoid

A figure can be drawn with a plane, a sphere, and a conicoid if the points in the diagram can be listed in an order $(P_1,P_2,\ldots,P_m)$ such that each point P_i can be drawn using the following three basic constructions:

(1) Construction POINT(P): takes a free point P in the space.

(2) Construction ON(P,o): takes a semi-free point P on a geometric object o.

(3) ConstructionINTERSECTION(P,O_1,O_2,O_3): takes the intersection P of three geometric objects O_1,O_2,O_3.

In the above constructions, O,O_1,O_2,O_3 are one of the three geometric objects: plane, sphere, and conicoid.

We can obtain other constructions on the basis of the three basic constructions. For instance, LINE(P,Q) is the line passing through points P and Q; PLINE(R,P,Q) is the line passing through point R and parallel to LINE(P,Q); PLANE(P,Q,R) is the plane passing through points P,Q, and R. SPHERE(O,r) is the sphere with center O and radius r; PSPHERE(O,P) is the sphere with center O and passing through point P; DSPHERE(P,Q) is the sphere whose radius is line PQ. These are parts of the constructions corresponding to lines, planes, and spheres. In addition, we introduce three constructions corresponding to conicoid: HYPERBOLOID(P,Q,d) is the hyperboloid of revolution consisting of the points R satisfying $||RP|-|RQ||=d$; ELLIPSOID(P,Q,d) is the ellipsoid of revolution consisting of the points R satisfying $|RP|+|RQ|=d$. PARABOLOID(PL,PO) is the paraboloid of revolution consisting of points R such that the distance between point R and plane PL is equal to the distance between point R and point PO.

In order to use conicoid as a new drawing tool, we need to introduce three new constraints:

SUM(R,P,Q,d): the sum of $|RP|$ and $|RQ|$ is a fixedvalue d, where R,P,Q are points.

DIF(R,P,Q,d): the difference of $|RP|$ and $|RQ|$ is a fixed value d, where R,P,Q are points.

EQU(R,PL,PO): the distance from R to PL is equal to the distance from R to PO, where R and PO are points and PL is a plane.

Let us assume that P and Q are fixed points. The locus of points R that satisfy the constraint **SUM**(R,P,Q,d) is an ellipsoid of revolution whose focal points are P and Q. The locus of points R that satisfy the constraint **DIF**(R,P,Q,d) is a hyperboloid of revolution whose focal points are P and Q. The locus of points R that satisfy the constraint **EQU**(R,PL,PO) is a paraboloid of revolution whose focal point is PO and directrix is PL. With these new predicates, we may use any constructive approach to find construction sequence for a 3D geometric constraint problem.

1.2 Algorithm

Since the work reported in this paper is closely related to the global propagation, we will give a brief introduction to this method. The global propagation approach[10] takes the declarative descriptions of a geometric diagram as input and tries to determine the position of a geometric object from not only the constraint involving the geometric object but also implicit information derived from other constraints. To determine the position of a point, first, find out the global information about this point; second, remove a constraint, and then, we would obtain a locus that this point is on; third, remove another constraint and we would obtain a curve that this point is on likewise. Hence we can determine the position of this point by computing the intersection of two loci. The global information needed in the propagation comes from a Geometric Information Base (*GIB*). The *GIB* for a configuration is a database containing all the properties of the configuration that can be deduced using a fixed set of geometric axioms. The details of the global propagation approach can be found in Ref.[10].

The global propagation approach in Ref.[10] is for 2D geometric constraint problem. In this paper, we will extend it to solve 3D geometric constraint problems. We define a 3D geometric constraint problem as follows

$$[[Q_1,\ldots,Q_m],[P_1,\ldots,P_n],[C_1,\ldots,C_m]],$$

where Q_i are the points whose construction order, $Q_1,\ldots,Q_m$ is given by the user; P_i are the points whose construction order will be determined by the program; and C_i are the constraints.

The proposed method takes a 3D geometric constraint problem as input, and outputs a construction sequence for it. In the following, we will list the main steps of the method.

1. For a constraint problem $[[Q_1,\ldots,Q_m],[P1,\ldots,P_n],[C_1,\ldots,C_m]]$ let $CT=[C_1,\ldots,C_m]$ be the constraint set, $CS=\emptyset$ the construction sequence set, $QS=[Q_1,\ldots,Q_m]$ the points with given construction order, and $PS=[P_1,\ldots,P_n]$ the rest points to be constructed. We assume that the problem is not over-constraint, i.e., we have $|CT| \le 3\times|PS|-6$.

2. Build the *GIB* described as before. Then repeat the following steps first for QS and then for PS until both QS and PS become empty.

3. Take a point P from QS or PS. With *GIB* we can decide the loci Lc_i of P satisfying $T \in CT$ involving P, assuming all points constructed in CS are known. We then obtain a set of triples: $\{(P,T_1,Lc_1),\ldots,(P,T_s,Lc_s)\}$. We consider three cases.

(1) $CS=\emptyset$. Point P is an arbitrarily chosen (free) point. We add a new construction CS=POINT(P) to CS.

(2) There exist $i \ne j \ne k$ such that $T_i \ne T_j \ne T_k$ and Lc_i, Lc_j, and Lc_k are not parallel planes or concentric spheres or conicoids having no intersection points. We add a new construction CS=INTERSECTION(P,Lc_i,Lc_j,Lc_k) to CS and remove T_i, T_j and T_k from CT.

(3) Point P is a semi-free point that can move freely on a plane, a sphere or a conicoid. We add a new construction CS=ON(P,Lc_i) to CS and remove T_i from CT.

4. Now we check whether the remaining problem is over-constrained, i.e., whether $|CT| > 3\times|PS|$

(1) If it is true, the construction sequence is invalid. If P is from QS, the order given by the user cannot be constructed and the method terminates. Otherwise, restore the removed constraints and repeat the preceding step for a new point from PS.

(2) If it is not true, point P is constructed. We need to repeat the preceding step for a new point.

The crucial step of the algorithm is how to determine the locus Lc in step 3. In the following, we will discuss this in detail. In this paper we only consider three kinds of loci: plane, sphere and conicoid. For the case of conicoid, the details can be found in Section 1.1.

For the case of plane, we first consider some partial properties for a plane P: $(P_1)P$ has a fixed direction; $(P_2)P$ passes through a known point; $(P_3)P$ is tangent to a known sphere. We thus have six possible forms to decide a plane: $(P_1,P_2),(P_1,P_3),(P_2,P_2,P_2),(P_2,P_2,P_3),(P_2,P_3,P_3),(P_3,P_3,P_3)$. Some basic situations can lead to the three cases. For example, the basic situation P is parallel to a known plane will lead to the case P_1. The basic situation there is a known point on P will lead to the case P_2. The basic situations (1) P is tangent to a known sphere; (2) P cuts a circle with a fixed radius in a known sphere will lead to the case P_3.

For the case of sphere, we similarly consider some partial properties for a sphere S: (S_1) The center of S is known; (S_2) The radius of S is known; (S_3) The center of S is on a known plane; (S_4) The center of S is on a known sphere; $(S_5)S$ passes through a known point; $(S_6)S$ is tangent to a known plane; $(S_7)S$ is tangent to a known sphere. Consider the geometric constraint problem in Example 1.

Example 1. Draw a plane passing through two known points A, B and cutting a circle whose radius is fixed on a known sphere O (See Fig.1). Since the plane cuts a circle whose radius is a fixed value on a known sphere O, by our above analysis the plane is tangent to a concentric sphere (the dotted sphere) of sphere O. We denote the concentric sphere by O' and the tangent point by O_1. Let r be the radius of sphere O, d the radius of the circle, then $|OO_1|=\sqrt{r^2-d^2}$, which is the radius of the sphere O'. Since the plane is tangent to the concentric sphere O', $OO_1 \perp AO_1$, $OO_1 \perp BO_1$. Points O, A, B are all known, thus point O_1 will be on the two spheres whose diameters are OA, OB respectively. Also, point O_1 lies on the concentric sphere O'. Subsequently, we can construct the point O_1 by INTERSECTION(O_1,O'), DSPHERE(O,A), DSPHERE(O,B). Now, the plane need to be constructed is PLANE (A,B,O_1).

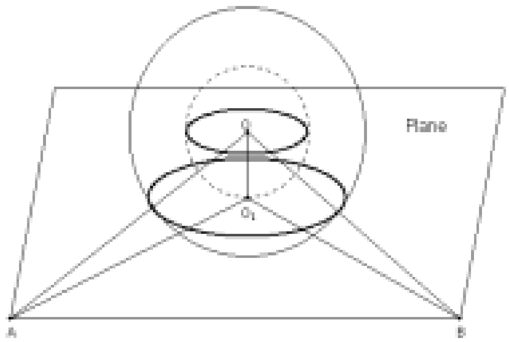

Fig.1 An example of geometric constraint problem

1.3 Solving 3D apollonius's drawing problem with conicoid

Let us show how to use the algorithm to solve the three-dimensional Apollonius's drawing problem. The geometric constraint problem is to determine a sphere satisfying condition (C_i,C_j,C_k,C_l) $(1\le i,j,k,l\le 3)$, where C_1: the sphere passes through a known point, C_2: the sphere is tangent to a known plane, C_3: the sphere is tangent to a known sphere.

There are altogether fifteen drawing problems. First, let us simply analyze the 3D Apollonius's drawing problem below. Determining a sphere needs two elements: the center and radius of sphere. If we know the center of sphere, the radius of sphere can be easily obtained by computing the distance from the center of sphere to the known point, the known plane or the point on the known sphere. The key step is to determine the center of sphere. Note that the sphere satisfies four constraints. If we ignore two constraints, the remaining two constraints would determine a surface that the center of sphere lies on. Hence, we will have six surfaces for the remaining two given constraints and the center of sphere will be determined by computing the intersection point of three surfaces. Subsequently, as long as we know surfaces determined by any two constraints, we will know the position of the center of sphere and solve the 3D Apollonius's drawing problem.

According to the three given constraints, there are altogether six cases (C_1,C_1),(C_1,C_2),(C_1,C_3),(C_2,C_2),(C_2,C_3),(C_3,C_3). The center of sphere that satisfy (C_i,C_j) will be a surface. We call the surface center surface. In the following, let us consider these cases in detail.

(C_1,C_1): The sphere passes through two known points. The center surface is the perpendicular bisector of the segment between the two points.

(C_1,C_2): The sphere passes through a known point and is tangent to a known plane. In this case, the distance from the center of sphere to the known point equals the distance from the center of circle to the known plane. According to the constraint **EQU**(*R*,*PL*,*PO*), the center surface is a paraboloid of revolution whose focal point is the known point and directrix is the known plane.

(C_2,C_2): The sphere is tangent to two known planes simultaneously. If two known planes are parallel, the center surface is a parallel plane equidistant with two known planes. Otherwise, it is the two planes passing through the intersection of the two known planes and the two angles respectively formed by the two planes and the known planes is equal.

(C_1,C_3): The sphere passes through a known point and is tangent to a known sphere. Different positions of the known point and sphere will result in different center surface. When the known point is outside the known sphere, the absolute value of the difference between the distance from the center of sphere to the known point and the distance from the center of sphere to the center of known sphere is a fixed value. Hence according to the constraint **DIF**(*R*,*P*,*Q*,*d*), the center surface is a hyperboloid of revolution whose focal points are the known point and the center of known sphere. When the known point is inside the known sphere, the absolute value of the sum of the distance from the center of sphere to the known point and from the center of sphere to the center of known sphere is a fixed value. Hence according to the constraint **SUM**(*R*,*P*,*Q*,*d*), the center surface is an ellipsoid of revolution whose focal points are the known point and the center of known sphere. When the known point is on the known sphere, the surface is the line passing through the known point and the center of the known sphere. This is a special case, since surface is a line. In the following, in order to be brief and clear, we only give the conclusion and corresponding constraint without analyzing.

(C_2,C_3): The sphere is tangent to a known plane and sphere simultaneously. When the known plane and the known sphere are intersecting, the center surface is a paraboloid of revolution whose focal point is the center of the known sphere and the directrix is a plane having fixed distance to the known plane. When the known plane is tangent to the known sphere, the center surface is the line perpendicular to the known plane and passing through the center of the known sphere or the paraboloid of revolution whose focal point is the center of the known sphere and the directrix is a plane having fixed distance to the known plane. When the known sphere and the known plane are separate, the center surface is the paraboloid of revolution whose focal point is the center of the known sphere and the directrix is a plane having fixed distance to the known plane.

(C_3,C_3): The sphere is tangent to two known spheres simultaneously. When the two known sphere are tangent and the unknown sphere and the two known spheres are tangent at the same point, the locus is a line passing through the center of two known spheres. When the two known spheres are interior tangent and the three spheres are not tangent at the same point, the center surface is an ellipsoid of revolution whose focal points are the centers of the two known spheres. When the two known spheres are intersecting and the unknown sphere is an inscribed sphere of one known sphere and a circumscribed sphere of another known sphere, the center surface is an ellipsoid of revolution whose focal points are the centers of the two known spheres. Except the cases above, the center surface is a hyperbola of revolution whose focal points are the centers of the two known spheres.

Example 2. Now let us give the construction sequence of one of Apollonius's drawing problems, the construction sequence of the others can be obtained similarly. Suppose the given constraints are (C_1,C_3,C_3,C_3). Based on the above analysis, the center of sphere is the intersection of three hyperbolas. Hence this problem has eight solutions, we only list one of the construction sequence, the other solutions can be obtained similarly. Assume that the known point is A, three centers of the known spheres are O_1,O_2,O_3. The radii respectively are R_1,R_2,R_3 that satisfy $R_1>R_2>R_3$. The spheres are all separate and the known point is outside the spheres. We essentially need one construction INTERSECTION(O), HYPERBOLOID(O_1,A,R_1), HYPERBOLOID(O_2,A,R_2), HYPERBOLOID(O_3,A,R_3) to find the center of sphere O. The radius of the sphere can be obtained through computing the distance from point O to the known point.

2 Scope of 3D Geometric Constraint Solving with Conicoid

Corresponding to the construction order of points in the diagram, there is a construction sequence $(CS_1,CS_2,\ldots,CS_m)$, where CS_i is one of the constructions defined in Section 2.1. Algebraically, each CS_i corresponds to a system of algebraic equations. Hence, a construction sequence corresponds to a system of algebraic equations. Since planes, spheres and conicoids are functions of points, we need only consider points. In the following, we will go into more detail about it.

First, let us consider the three basic constructions. If P is introduced by POINT(P), and P is a free point, no equation is introduced. If P is introduced by ON(P,O), we need one algebraic equation, $f(u,v,x)=0$ to describe P, where u, v are parameters and x is a variable. For given u and v, we can compute x from $f(u,v,x)=0$. Hence $f(u,v,x)=0$ will cause infinite solutions. The construction INTERSECTION(P,O_1,O_2,O_3) will produce a system of algebraic equations:

$$\begin{cases} f_1(x_1,x_2,x_3)=0 \\ f_2(x_1,x_2,x_3)=0, \\ f_3(x_1,x_2,x_3)=0 \end{cases}$$

where $f_i(x_1,x_2,x_3)=0$ $(i=1,2,3)$ are the equations of the three geometric objects respectively.

Let FS be the system of algebraic equations produced by the construction sequence $(CS_1,CS_2,\ldots,CS_m)$. Based on the Wu-Ritt's zero decomposition algorithm[14], FS can be transformed into a system of algebraic equations TS in triangular form :

$$\begin{cases} F_1(u_1,\ldots,u_q,x_1)=0 \\ F_1(u_1,\ldots,u_q,x_1,x_2)=0 \\ \ldots \\ \ldots \\ F_1(u_1,\ldots,u_q,x_1,\ldots,x_{kl})=0. \end{cases}$$

Since the variables are introduced at most three by three, by the Bezout Theorem, $Degree(F_i)\le 8(1\le i\le l)$.

We have proved that a construction sequence of planes, spheres and conicoids will lead to a triangular set of polynomials with degree less than nine. Now, we will prove that each triangularised polynomial equation with degree less than nine can be generalized by a construction sequence of planes, spheres and conicoids. Note that we need only solve a univariate equation with degree less than nine in the process of solving TS. Hence we only need to prove that the root of any univariate equation with degree less than nine can be constructed with planes, spheres and conicoids.

Theorem. Any points whose dimensions are the roots of univariate equations of degree less than nine can be constructed with planes, spheres, and conicoids.

Proof. Here, we only give the proof of the case of degree eight. The other case can be similarly proved. Let the equation be

$$p(x)=x^8+ax^7+bx^6+cx^5+dx^4+ex^3+fx^2+gx+h,$$

where a, b, c, d, e, f, g, h are numerical coefficients. We can construct three conicoids as follows:

$$f_1(x,y,z)=xy-1, \tag{1}$$

$$f_2(x,y,z)=yz-(x-c_2)^2, \tag{2}$$

$$f_3(x,y,z)=(x-1)^2+c_1y^2+z_2+c_3xy+c_4yz+c_5xz+c_6x+c_7y+c_8z, \tag{3}$$

by equation (1), we have

$$y=1/x, \tag{4}$$

by equations (2) and (4), we have

$$z=x^3-2c_2x^2+c^2{}_2x^2, \tag{5}$$

substituting y,z for equations (4) and (5) into equation (3), we obtain

$$p(x)=x^8-4c_2x^7+(c_5+6c_2^2)x^6+(c_8-3c_5c_2-4c_2^3)x^5+(1+c_2^4-2c_8c_2+c_4+3c_5c_2^2)x^4+$$
$$(c_6-c_5c_2^3-2c_2c_4+c_8c_2^2)x^3+(c_3-c_6c_2+c_4c_2^2)x^2+(c_7-c_3c_2)x+c_1.$$

Comparing the coefficients of $q(x)$ with that of $p(x)$, clearly, the roots of $p(x)$ can be constructed by three conicoid:

$$xy-1=0,$$

$$yz-(x-\frac{1}{4}a)^2=0,$$

$$(x-1)^2+hy+z^2+(f-\frac{1}{4}ae+\frac{1}{256}a^4b-\frac{3}{4096}a^6+\frac{1}{64}a^3c-\frac{1}{16}a^2)xy+(b-\frac{3}{8}a^2)xz+$$
$$(d+\frac{3}{16}a^2b-\frac{11}{256}a^4-\frac{1}{2}ac-1)yz+(e-\frac{1}{16}a^3b+\frac{7}{512}a^5-\frac{1}{2}ad+\frac{3}{16}a^2c+\frac{1}{2}a)x+$$
$$(g-\frac{1}{4}af+\frac{1}{16}a^2e-\frac{1}{1024}a^5b+\frac{3}{16384}a^7-64a^3d+\frac{1}{256}a^4c+\frac{1}{64}a^3)y+(c-\frac{3}{4}ab+\frac{7}{32}a^3)z=0.$$

3 Conclusion

In this paper, by introducing conicoid as a new drawing tool, we broaden the solvable scope of GCS with constructive approaches. With planes and spheres only, we can draw a figure that can be described by a sequence of equations of degree less than three. After adding conicoid, we can draw a figure that can be described by a sequence of equations of degree of less than nine. Moreover, note that to draw a figure with planes, spheres, and conicoids, we need to solve a sequence of equations of degree less than nine. Conversely, any sequence of equations of degree less than nine can be obtained by a construction sequence of planes, spheres, and conicoids.

References:

[1] Lee, J.Y., Kim, K. Geometric reasoning for knowledge-based design using graph representation. Computer-Aided Design, 1998,28(10):831~841.

[2] Sunde, G. Specification of shape by dimension and other geometric constraints. Geometric Modeling for CAD Application. North-Holland, 1998. 199~213.

[3] Verroust, A., Schonek, F., Roller, D. Rule-Oriented method for parameterized computer-aided design. Computer-Aided Design, 1992,24(10):531~540.

[4] Bruderlin, B. Constructing three-dimensional geometric objects defined by constraints. In: Workshop on Interactive 3D Graphics. ACM, 1986. 111~129.

[5] Hoffmann, C. Geometric constraint solving in R^2 and R^3. In: Du, D.Z., Huang, F., eds. Computing in Euclidean Geometry. Word Scientific, 1995. 266~298.

[6] Kramer, G.A. Solving Geometric Constraints Systems: a Case Study in Kinematics. MA: MIT Press, 1992.

[7] Kondo, K. Algebraic method for manipulation of dimensional relationships in geometric models. Computer-Aided Design, 1992,24(3):141~147.

[8] Lin, V.C., Gossard, D.C., Light, R.A. Variational geometry in computer-aided design. Computer Graphics, 1981,15(3):171~177.

[9] Owen, J. Algebraic solution for geometry from dimensional constraints. In: ACM Symposium Found of Solid Modeling. ACM Press, Austin Tx, 1991. 397~407.

[10] Gao, X.S., Chou, S.C. Solving geometric constraint systems I, a global propagation approach. Computer-Aided Design, 1998,30(1):47~54.

[11] Vermeer, P. A spatial constraint problem. In: Merlet, J.P., Ravani, B., eds. Proceedings of the Computational Kinematics'95. Kluwer Academy Publisher, 1995. 83~92.

[12] Durand, C., Hoffmann, C.M. A systematic framework for solving geometric constraints analytically. Technical Report, Department of Computer Science, Purdue University, 2000.

[13] Hoffmann, C. Yuan, B. Spatial constraint solving approaches. Technical Report, Department of Computer Science, Purdue University, 1999.

[14] Wu, Wentsün. Basic Principles of Mechanical Theorem Proving in Geometries. Springer-Verlag, 1993.

用圆锥曲面求解几何约束问题

蒋　鲲[1,2]，高小山[1]

[1](中国科学院　数学与系统科学研究院,北京　100080);

[2](黑龙江大学　理学院,黑龙江　哈尔滨　150080)

摘要: 通常大多数三维参数化 CAD 系统都只用平面和球面作为最基本的作图工具,这在某种程度上限制了三维参数化 CAD 系统的作图范围.通过引进一类新的作图工具,使得三维参数化 CAD 系统的作图范围得到扩大.同时证明了一个三维几何图形可以用平面、球面和圆锥曲面构造出来的充分必要条件是这个三维几何图形可以用一个三角化的次数小于 9 的代数方程组来描述.通过引进圆锥曲面作为新的作图工具,著名的三维 Appolonius 作图问题可以被完全求解.

关键词: 几何约束求解;圆锥曲面;参数化 CAD

中图法分类号: TP391　　　文献标识码: A