



Fig.2 Curves of convergence rates of six MOEAs based on nine MOPs

图2 6种算法在9个较难测试函数上的收敛速度曲线

3.3 进一步讨论

通过上文的实验验证了 HMOFA 算法总体上具有显著的收敛性、多样性和鲁棒性优势,表明了混合水平正交实验初始化种群的方法、利用外部档案的精英个体引导个体移动的策略以及 3 点最短路径的修剪技术有机结合,协同地提高了算法的整体性能.为进一步检验在 HMOFA 算法中引入的混合水平正交初始化方法和档案精英个体引导萤火虫移动策略的有效性,分别设计实验 3 和实验 4 加以测试.

实验 3.为检验 HMOFA 算法中混合水平正交初始化策略的有效性,这里将 HMOFA 算法和未使用混合水平正交初始化方法而仅使用随机初始化方法的 HMOFA 算法(简记为 HMOFA(r))一同在 8 个较难的测试问题上进行 HV 性能测试.需要指出的是,实验 3 中 HMOFA 和 HMOFA(r)除了使用的初始化种群方法不同之外,这两种算法的其他方面均是相同的.表 6 给出了 HMOFA 算法和 HMOFA(r)算法在 8 个测试函数上获得的 HV 值.

从表 6 可以看出,HMOFA 算法在 8 个较难测试问题上获得了全部最优的 HV 均值,而 HMOFA(r)算法仅在 ZDT6 和 DTLZ1 问题上获得了与 HMOFA 算法相同的最优的 HV 均值,而 HMOFA(r)在其他 6 个测试问题上获得的 HV 均值都不如 HMOFA 算法的好.另外,从表 6 的 t -检验结果来看,HMOFA 算法对比 HMOFA(r)算法在 HV 性能上的净胜得分为 6.由于 HV 指标能够同时反映解群的收敛性和分布性,因此,采用混合水平正交初始化

种群的方法有利于算法改善群体的收敛性与分布性.由此表明了混合水平正交初始化种群的方法是有效的.究其原因,混合水平正交初始化方法一方面产生均匀分布于整个搜索空间的初始解点,初始群体的多样性促进了种群在全局范围内寻优而不易于陷入局部最优.另外,由于混合水平正交初始化方法一般能产生接近用于指定规模的初始群体,可高效地利用计算资源,这些因素相互作用改善了算法的性能.

Table 6 Results of HV for HMOFA and HMOFA(*r*) on eight MOPs

表 6 HMOFA 和 HMOFA(*r*)在 8 个测试函数上获得的 HV 值

函数		HMOFA	HMOFA(<i>r</i>)
ZDT4	Mean	6.61E-01	0.00E+00
	Std.	4.20E-04	0.00E+00
	<i>t</i> -test		+
ZDT6	Mean	4.06E-01	4.06E-01
	Std.	6.50E-05	7.10E-05
	<i>t</i> -test		=
DTLZ1	Mean	0.00E+00	0.00E+00
	Std.	0.00E+00	0.00E+00
	<i>t</i> -test		=
DTLZ2	Mean	4.10E-01	4.09E-01
	Std.	2.60E-03	4.00E-03
	<i>t</i> -test		+
DTLZ3	Mean	1.00E+00	9.99E-01
	Std.	1.90E-07	5.10E-04
	<i>t</i> -test		+
DTLZ6	Mean	9.47E-02	0.00E+00
	Std.	1.10E-05	0.00E+00
	<i>t</i> -test		+
DTLZ7	Mean	3.17E-01	3.04E-01
	Std.	3.00E-03	6.20E-03
	<i>t</i> -test		+
Viennet3	Mean	8.42E-01	8.41E-01
	Std.	1.00E-04	2.10E-04
	<i>t</i> -test		+
	Better(+)		6
	Same(=)		2
	Worse(-)		0
	Score		6

实验 4.为检验 HMOFA 算法中档案精英个体引导萤火虫移动机制的有效性,这里将 HMOFA 和未采用精英解引导萤火虫移动而仅使用基本 FA 算法萤火虫移动方法的 HMOFA 算法(简记为 HMOFA(*b*))一同在 8 个测试函数上进行 HV 性能测试,实验 4 中,HMOFA 和 HMOFA(*b*)除使用的萤火虫移动机制不同之外,这两种算法的其他方面均是一致的.表 7 给出了 HMOFA 算法和 HMOFA(*b*)算法在 8 个测试函数上获得的 HV 值.

从表 7 可以看出,HMOFA 在 8 个较难问题上获得了全部最优的 HV 均值,而 HMOFA(*b*)算法仅在 DTLZ1 问题上获得与 HMOFA 算法相同的最优的 HV 均值,而在其他 7 个 MOP 问题上获得的 HV 均值都要差于 HMOFA 算法.表 7 的 *t*-检验结果中,HMOFA 算法对比 HMOFA(*b*)算法在 HV 性能上的净胜分为 7,而仅在 DTLZ1 问题上与 HMOFA(*b*)算法的结果无显著性区别.究其原因,HMOFA 算法利用档案精英个体指导搜索,精英解携带了与问题相关的有益信息,使得算法能够较快地收敛.另外,由于 HMOFA 算法采用随机方式从档案中选取精英解引导个体移动,因而不会导致群体在局部区域聚集.所以,这里的档案精英解引导策略有利于改善算法的收敛性与多样性.实验 4 的结果表明,档案精英解指导种群搜索的方法是有效的.

Table 7 Results of HV for HMOFA and HMOFA(*b*) on eight MOPs

表 7 HMOFA 和 HMOFA(*b*)在 8 个测试函数上的 HV 值

函数		HMOFA	HMOFA(<i>b</i>)
ZDT4	Mean	6.61E-01	0.00E+00
	Std.	4.20E-04	0.00E+00
	<i>t</i> -test		+
ZDT6	Mean	4.06E-01	0.00E+00
	Std.	6.50E-05	0.00E+00
	<i>t</i> -test		+

Table 7 Results of HV for HMOFA and HMOFA(*b*) on eight MOPs (Continued)
 表 7 HMOFA 和 HMOFA(*b*)在 8 个测试函数上的 HV 值(续)

函数		HMOFA	HMOFA(<i>b</i>)
DTLZ1	Mean	0.00E+00	0.00E+00
	Std.	0.00E+00	0.00E+00
	<i>t</i> -test		=
DTLZ2	Mean	4.10E-01	1.79E-01
	Std.	2.60E-03	1.80E-02
	<i>t</i> -test		+
DTLZ3	Mean	1.00E+00	9.20E-01
	Std.	1.90E-07	1.60E-02
	<i>t</i> -test		+
DTLZ6	Mean	9.47E-02	0.00E+00
	Std.	1.10E-05	0.00E+00
	<i>t</i> -test		+
DTLZ7	Mean	3.17E-01	0.00E+00
	Std.	3.00E-03	0.00E+00
	<i>t</i> -test		+
Viennet3	Mean	8.42E-01	7.90E-01
	Std.	1.00E-04	1.60E-02
	<i>t</i> -test		+
	Better(+)		7
	Same(=)		1
	Worse(-)		0
	Score		7

4 结 论

现实中的多目标优化问题不断增多且日益复杂,迫切需要发展新的有效的多目标优化方法应对挑战.近年来,基于新型进化范例和混合机制的多目标优化算法渐已成为多目标优化领域的新的研究热点,它们在求解各种复杂 MOP 问题中占据重要地位.本文提出一种混合型多目标萤火虫算法 HMOFA,该算法将基本 FA 算法拓展至多目标优化领域,并将混合水平正交初始化方法、档案精英个体引导策略和 3 点最短路径技术融入其中,以提高算法的整体性能.实验结果表明,HMOFA 算法在收敛性、多样性和鲁棒性等方面要显著地优于其他 5 种对比算法.未来将从 3 个方面做进一步的工作:(1) 利用更多、更困难的 MOP 问题测试 HMOFA 算法;(2) 将 HMOFA 算法应用于工程实践中的 MOP 问题;(3) 尝试将新的学习机制引入 MOEA 算法,不断改善 MOEA 算法求解复杂 MOP 问题的性能.

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