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RCC11 复合表的表示*

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Representation of RCC11 Composition Table

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Abstract: This paper is mainly concerned with the relation-algebraic aspects of the well-known Region Connection Calculus (RCC). It is shown that the complemented closed disk algebra is a representation for the relation algebra determined by the RCC11 table, which was first described by Düntsch. The domain of this algebra contains two classes of regions, the closed disks and closures of their complements in the real plane, and the contact relation is the standard Whiteheadean contact (i.e. $a\mathbf{C}b$ iff $a\cap b\neq\varnothing$).
Key words: region connection calculus; contact relation algebras; RCC11 composition table; complemented closed disk algebra; dual-relation set; extensionality

摘 要: 主要研究熟知的区域连接演算(region connection calculus,简称 RCC)的关系代数方面的性质.证明了补闭圆盘代数恰好构成 RCC11 复合表的一个表示,其中,RCC11 复合表是由 Düntsch 于 1999 年引入的.补闭圆盘代数由两类区域构成:一类是实平面中的所有闭圆盘;另一类是实平面中的所有闭圆盘的补的闭包组成.而连接关系为经典的 Whiteheadean 连接,即对区域 $a,b,a\mathbf{C}b$(表示 a,b 有连接关系)当且仅当 $a\cap b\neq\varnothing$.
关键词: 区域连接演算;连接关系代数;RCC11 复合表;补闭圆盘代数;对偶关系集;扩张性
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1 Introduction

Qualitative spatial reasoning (QSR) is an important subfield of **AI** which is concerned with the qualitative aspects of representing and reasoning about spatial entities. A large part of contemporary qualitative spatial reasoning is based on the behavior of “part of ” and “connection” (or “contact”) relations in various domains[1,2], and the expressive power, consistency and complexity of relational reasoning has become an important object of study in QSR.

Rather than giving attention to all the various systems existing in the market, we shall focus on one of the most widely referenced formalism for QSR, the Region Connection Calculus (RCC). RCC was initially described by Randell, Cohn and Cui in Ref.[3], which is intended to provide a logical framework for incorporating spatial reasoning into **AI** systems.

In the RCC theory, the Jointly Exhaustive and Pairwise Disjoint (JEPD) set of topological relations known as RCC8 are identified as being of particular importance. RCC8 contains relations: “*x* is disconnected from *y*”,“*x* is externally connected to *y*”, “*x* partially overlaps *y*”, “*x* is a equal to *y*”, “*x* is a tangential proper part of *y*”, “*x* is a non-tangential proper part of *y*”, “*x* is a non-tangential proper part of *y*”, and the inverses of the latter two relations. Interestingly, this classification of topological relations has been independently given by Egenhofer[4] in the context of Geographical Information Systems (GIS). Since RCC8 is JEPD, it supports a composition table. The RCC8 composition table appears first in Ref.[5] and coincides with that of Ref.[4].

Originating in Allen’s analysis of temporal relations[6,7], the notion of a composition table (**CT**) has become a key technique in providing an efficient inference mechanism for a wide class of theories. Generally speaking, a **CT** is just a mapping $\mathbf{CT}\ \mathbf{Rels}\times\mathbf{Rels}\rightarrow 2^{\mathrm{Rels}}$, where **Rels** is a set of relation symbols[8]. For three relation symbols **R**, **S** and **T**, we say $\langle\mathbf{R},\mathbf{T},\mathbf{S}\rangle$ is a composition triad in **CT** if **T** is in **CT** (**R**,**S**). A model of **CT** is then a pair $\langle U,v\rangle$, where U is a set and v is a mapping from **Rels** to the set of binary relations on U such that $\{v(\mathbf{R}):\mathbf{R}\in\mathbf{Rels}\}$ is a partition of $U\times U$ and $v(\mathbf{R})\circ v(\mathbf{S})\subseteq\cup_{\boldsymbol{T}\in CT(\mathbf{R},\mathbf{S})}v(\mathbf{T})$ for all $\mathbf{R},\mathbf{S}\in\mathbf{Rels}$, where $\circ$ is the usual relation composition. A model $\langle U,v\rangle$ is called consistent if $\boldsymbol{T}\in CT(\mathbf{R},\mathbf{S})\Leftrightarrow(v(\mathbf{R})\circ v(\mathbf{S}))\cap v(\mathbf{T})\neq\varnothing$ for all $\mathbf{R},\mathbf{S},\mathbf{T}\in\mathbf{Rels}$[9]. We call a consistent model extensional if $v(\mathbf{R})\circ v(\mathbf{S})\subseteq\cup_{\boldsymbol{T}\in CT(\boldsymbol{R},\mathbf{S})}v(\mathbf{T})$ for all $\mathbf{R},\mathbf{S}\in\mathbf{Rels}$[9]. Note if a **CT** has an extensional model $\langle U,v\rangle$, then by a theorem given in Ref.[10], this **CT** is the composition table of a relation algebra and $\langle U,v\rangle$ is a representation of this relation algebra. In what follows, when the interpretation mapping v is clear from the context, we also write U for this model.

To obtain an extensional model of the RCC8 **CT**, one should restrict the domain of possible regions: an RCC model might contain too much regions. Düntsch[5] has shown that the domain of closed disks of the Euclidean plane provides an extensional model of the RCC8 **CT**, namely, the relation algebra determined by the RCC8 **CT** can be represented by the closed disk algebra. One serious problem with these regions is that they are not closed under complementation. But, as noted by Stell[11], complement is a fundamental concept in spatial relations. These regions are therefore too restrictive. In Ref.[8], with modeling complementation in mind, Düntsch refines RCC8 to RCC11. The RCC11 **CT** is also given and it “turns out that there is a relation algebra *A* whose composition is represented by the RCC11 table. *A*, however, cannot come from an RCC model as Proposition 8.6 shows, and no representation of *A* is known”[8].

The main contribution of this paper is to provide an extensional model for the RCC11 **CT**. Note models of the RCC11 **CT** are closed under complementation. Our model then contains simply two kinds of regions: the closed disks and the closures of their complements in the Euclidean plane, where two regions are connected if they have nonempty intersection. Note this domain is clearly a sub-domain of the standard RCC model associated to R^2. We then have two methods to introduce the RCC11 relations on this domain: the first system of relations is obtained by restriction of the RCC11 relations in the standard RCC model associated to R^2, the second can be defined by the connectedness relation on this domain. Interestingly these two systems of relations are identical. The binary relation algebra generated by the connectedness relation, the complemented closed disk algebra, has 11 atoms that correspond to the RCC11 relation and the composition of this algebra is just the one specified by the RCC11 **CT**. Note that hand building of composition tables even for a small number of relations is an arduous and tedious work. Although there are more general methods to compute composition tables (see e.g.Ref.[12]), these methods seem not appropriate for the present purposes. Our requirements are manifold: the method should be applicable not only for determining the composition table, but also for checking the consistency and extensionality of the table. To this aim, we propose a specialized approach to reduce the calculations: by using this approach, the work needed can be reduced to nearly 1/8 of that needed by the cell-by-cell verification.

In a word, we answer a problem posed by Düntsch in Ref.[8], about the representation of RCC11 **CT**, and introduce a specified method to reduce the calculation of RCC **CT**. The relation-algebraic aspects of RCC is riched, and the application of RCC theory to the regions with complement is extended.

The rest of the paper is arranged as follows. In next section, we briefly summarize some basic concepts of contact relation algebras and the RCC theory. The notions of dual relation set and dual generating set for RCC relations are introduced in Section 3. Based on these notions, a very effective approach to determining the RCC weak **CT** is introduced. Section 4 introduces the complemented closed disk algebra L which is a representation of the relation algebra determined by the RCC11 composition table. Summary and outlook are given in the last section.

2 Contact Relation Algebras

In this section we summarize some basic concepts of contact relation algebras and the RCC models. For contact relation algebras our references are Ref.[8,13–15], and for RCC models[2,3,16–18].

Recall in a relation algebra (RA) (A,+,·,–,0,1,o,~,1′), (A,+,·,–,0,1) is a Boolean algebra, and (A, o,~,1′) is a semigroup with identity 1′, and $a^{\breve{}\breve{}}=a,(a\circ b)^{\breve{}}=b^{\breve{}}\circ a^{\breve{}}$. In the sequel, we will usually identify algebras with their base set.

An important example of relation algebra is the full algebra of binary relations on the underlying set U, written ($Rel(U)$,∪,∩,–,∅,U×U,∘,~,1′), where $Rel(U)$ is the set of all binary relations on U, ∘ is the relational composition, ~ the relation converse, and 1′ is the identity relation on U. For **R**∈$Rel(U)$, and $x,y,z\in U$ we usually write x**R**y or **R**(x,y) if (x,y)∈**R**.

Recall a subset A of $Rel(U)$ which is closed under the distinguished operations of $Rel(U)$ and contains the distinguished constants is called an algebra of binary relations (BRA) on U A relation algebra A is called representable if it is isomorphic to a subalgebra of a product of full algebras of binary relations, A is called integral, if 1′ is an atom of A.

To avoid trivialities, we always assume that the structures under consideration have at least two elements. Suppose that U is a nonempty set of regions, and that **C** is a binary relation on U which satisfies

(C1) **C** is reflexive and symmetric,

(C2) $(\forall x,y\in U)[x=y\leftrightarrow\forall z\in U(\mathbf{C}(x,z)\leftrightarrow\mathbf{C}(y,z))]$.

Düntsch *et al*.[13] call a binary relation **C** which satisfies (C1) and (C2) a contact relation; and an RA generated by a contact relation will be called a contact RA(CRA). A contact relation **C** on an ordered structure ⟨U,≤⟩ is said to be compatible with ≤ if –(**C**∘–**C**)=≤. In this paper we only consider compatible contact relations on orthocomplemented lattices. Recall an orthocomplmented lattice is a bounded lattice ⟨L,0,1,∨,∧⟩ equipped with a unary complemented operation $':L\to L'$ such that $x''=x$, $x\wedge x'=0$, $x\le y\Leftrightarrow x'\ge y'$.

Suppose L is an orthocomplemented lattice containing more than four elements and **C** is a contact relation other than the identity. Set $U=L\backslash\{0,1\}$. Since 1_U is RA definable[8], we can restrict the contact relations **C** and other relations definable by **C** on U. The following relations can then be defined from **C** on U:

DC	=	$-\mathbf{C}$	**P**	=	$-(\mathbf{C}\circ-\mathbf{C})$
$1'$	=	$\mathbf{P}\cdot\mathbf{P}\sim$	**PP**	=	$\mathbf{P}-1'$
O	=	$\mathbf{P}\sim\circ p$	**PO**	=	$\mathbf{O}\cdot-(\mathbf{P}+\mathbf{P}\sim)$
EC	=	$\mathbf{C}\cdot-\mathbf{O}$	**TPP**	=	$\mathbf{PP}\cdot(\mathbf{EC}\circ\mathbf{EC})$
NTPP	=	$\mathbf{PP}\cdot-\mathbf{TPP}$	#	=	$-(\mathbf{P}+\mathbf{P}\sim)$
T	=	$-(\mathbf{P}\circ\mathbf{P}\sim)$	**PON**	=	$\mathbf{P}\cdot\#\cdot-\mathbf{T}$
POD	=	$\mathbf{O}\cdot\#\cdot\mathbf{T}$	**ECD**	=	$-\mathbf{O}\cdot\mathbf{T}$
ECN	=	$\mathbf{EC}\cdot-\mathbf{ECD}$	**PODZ**	=	$\mathbf{ECD}\circ\mathbf{NTPP}$
DN	=	$\mathbf{DR}-\mathbf{ECD}$	**PODY**	=	$\mathbf{POD}-\mathbf{PODZ}$

We have the following systems of JEPD relations on U[8]:

RCC5 relations: $\mathcal{R}_5=\{1',\mathbf{PP},\mathbf{PP}^{\sim},\mathbf{PO},\mathbf{DR}\}$;

RCC7 relations: $\mathcal{R}_7=\{1',\mathbf{PP},\mathbf{PP}^{\sim},\mathbf{PON},\mathbf{POD},\mathbf{ECD},\mathbf{DN}\}$;

RCC8 relations: $\mathcal{R}_8=\{\mathbf{DC},\mathbf{EC},\mathbf{PO},1',\mathbf{TPP},\mathbf{NTPP},\mathbf{TPP}^{\sim},\mathbf{NTPP}^{\sim}\}$;

RCC11 relations: $\mathcal{R}_{11}=\{1',\mathbf{TPP},\mathbf{TPP}^{\sim},\mathbf{NTPP},\mathbf{NTPP}^{\sim},\mathbf{PON},\mathbf{PODY},\mathbf{PODZ},\mathbf{ECN},\mathbf{ECD},\mathbf{DC}\}$.

We summarize some characterizations of these RCC relations.

Lemma 2.1. Suppose L is an orthocomplemented lattice L with $|L|>4$ and **C** is a compatible contact relation on L other than the identity. Then for any $x,y\in U=L\backslash\{0,1\}$ we have the following resuls:

(1) $x\mathbf{PON}y$ iff $x\wedge y>0$, $x\vee y<1$, $x\wedge y'>0$ and $x'\wedge y>0$;

(2) $x\mathbf{POD}y$ iff $x\wedge y>0$, $x\vee y=1$;

(3) $x\mathbf{PP}y$ iff $x<y$;

(4) $x\mathbf{ECD}y$ iff $x=y'$;

(5) $x\mathbf{ECN}y$ iff $x<y'$ and $x\mathbf{C}y$;

(6) $x\mathbf{TPP}y$ iff $x<y$ and $x\mathbf{C}y'$;

(7) $x\mathbf{NTPP}y$ iff $x<y$ and $x\mathbf{DC}y'$;

(8) $x\mathbf{PODY}y$ iff $y'<x$ and $x'\mathbf{C}y'$;

(9) $x\mathbf{PODZ}y$ iff $y'<x$ and $x'\mathbf{DC}y'$.

In what follows, we shall often write respectively $-x,x+y,x-y$ for x', $x\vee y$ and $x\wedge y'$.

2.1 Models of the RCC axioms

The Region Connection Calculus (RCC) was originally formulated by Randel, Cui and Cohn[3]. There are several equivalent formulations of RCC[8,16], we adopt in this paper the one in terms of Boolean connection algebra (BCA)[16].

Definition 2.1. A model of the RCC is a structure $\langle A,\mathbf{C}\rangle$ such that

A1. $A=\langle A;0,1,',\vee,\wedge\rangle$ is a Boolean algebra with more than two elements.

A2. **C** is a symmetric and reflexive binary relation on $A\backslash\{0\}$.

A3. $\mathbf{C}(x,x')$ for any $x\in A\backslash\{0,1\}$.

A4. $\mathbf{C}(x,y\vee z)$ iff $\mathbf{C}(x,y)$ or $\mathbf{C}(x,z)$ for any $x,y,z\in A\backslash\{0,1\}$.

A5. For any $x\in A\backslash\{0,1\}$, there exists some $w\in A\backslash\{0,1\}$ such that $\mathbf{C}(x,w)$ doesn't hold.

Stell[16] calls such a construction a Boolean connection algebra (BCA), and this conception is stronger than the Boolean contact algebra given by Düntsch[5].

In particular, the connection in a BCA satisfies Condition (C2) and hence is a contact relation in Düntsch's sense.

Given a regular connected space X, write $RC(X)$ for the regular closed algebra of X. Then with the standard Whiteheadean contact (i.e. $a\mathbf{C}b$ iff $a\cap b\neq\varnothing$), $\langle RC(X),\mathbf{C}\rangle$ is a model of the RCC[19]. These models are called standard

RCC models[8]. Later we shall refer the standard model associated to a regular connected space X simply $RC(X)$.

If an RCC model A satisfies the following interpolation property (INT for short)

$$x\mathbf{NTPP}y\rightarrow\exists z(x\mathbf{NTPP}z\wedge z\mathbf{NTPP}y)$$

We call it a strong RCC model. Standard RCC models associated to $\mathcal{R}^n$ are strong models.

3 Dual Relation Sets and RCC Composition Tables

In this section we shall propose a specialized approach for reducing the computational work of establishing an RCC **CT**. This approach can also be applied in determining the consistency and extensionality of an RCC **CT**.

3.1 Dual relation set and dual generating set

Definition 3.1. Let $\langle L,'\rangle$ be an orthocomplemented lattice with $|L|>4$ and let $U=L\backslash\{0,1\}$. For two relations **R**, **S** on U, if $(\forall x,y\in U)x\mathbf{S}y\leftrightarrow x\mathbf{R}y'$, then **S** is called the right dual of **R** and is denoted by $\mathbf{R}^d$. If $(\forall x,y\in U)x\mathbf{S}y\leftrightarrow x\mathbf{R}y'$, then **S** is called the right dual of **R** and is denoted by $\mathbf{R}^d$. If $(\forall x,y\in U)x\mathbf{S}y\leftrightarrow x'\mathbf{R}y$, then we call **S** the left dual of **R** and denote it by ${}^d\mathbf{R}$.

The right dual and the left dual are just two unitary operations on $Rel(U)$. For any $X\subseteq Rel(U)$, we call the relation set X a dual relation set on U if X is closed under the right dual and the left dual. Clearly $Rel(U)$ itself is a dual relation set on U, and intersection of dual relation sets on U is also dual on U. We define the dualization of a relation set X, denoted by $d(X)$, to be the least dual relation set containing X as a subset. For a dual relation set $\mathcal{R}$, we can find a minimal subset $\mathcal{S}$ of $\mathcal{R}$ such that $\mathcal{R}=\mathcal{S}\cup\mathcal{S}^d={}^d\mathcal{S}\cup\mathcal{S}$. We call $\mathcal{S}$ a dual generating set of $\mathcal{R}$.

The following propositions summarize some basic properties of these two dual operations and can be easily checked.

Lemma 3.1. Let $\langle L,'\rangle$ be an orthocomplemented lattice with $|L|>4$ and let $U=L\backslash\{0,1\}$. Suppose **R**, **S** are two relations on U. Then the following conditions hold:

(1) $\mathbf{R}^d=\mathbf{R}\circ\mathbf{ECD}$, ${}^d\mathbf{R}=\mathbf{ECD}\circ\mathbf{R}$; (2) $\mathbf{R}^{dd}=\mathbf{R}$, ${}^{dd}\mathbf{R}=\mathbf{R}$, ${}^d(\mathbf{R}^d)=({}^d\mathbf{R})^d$;

(3) $\mathbf{R}^{\breve{}d\breve{}}={}^d\mathbf{R}$, $({}^d(\mathbf{R}^{\breve{}}))^{\breve{}}=\mathbf{R}^d$; (4) $\mathbf{R}^d\cap\mathbf{S}\neq\varnothing$ iff $\mathbf{R}\cap\mathbf{S}^d\neq\varnothing$;

(5) ${}^d\mathbf{R}\cap\mathbf{S}\neq\varnothing$ iff $\mathbf{R}\cap\mathbf{S}^d\neq\varnothing$; (6) For all $x,y\in U$, $(x,y)\in{}^d(\mathbf{R}^d)$ iff $(x',y')\in\mathbf{R}$.

Theorem 3.1. Let $\langle L,'\rangle$ be an orthocomplemented lattice with $|L|>4$ and let $U=L\backslash\{0,1\}$. Suppose **C** is a compatible contact relation of U other than the identity and $\mathcal{R}$ is a JEPD set of relations in the CRA of U. Then for any $\mathbf{M},\mathbf{N}\in\mathcal{R}$, we always have the following equations, where $\circ_\omega$ denotes the weak composition, namely $\mathbf{M}\circ_\omega\mathbf{N}=\cup\{\mathbf{R}\in\mathcal{R}: \mathbf{R}\cap\mathbf{M}\circ\mathbf{N}\neq\varnothing\}$:

(1) $(\mathbf{M}\circ\mathbf{N})^{\breve{}}=\mathbf{N}^{\breve{}}\circ\mathbf{M}^{\breve{}}$;

(2) $(\mathbf{M}\circ\mathbf{N})^d=\mathbf{M}\circ\mathbf{N}^d$, ${}^d(\mathbf{M}\circ\mathbf{N})={}^d\mathbf{M}\circ\mathbf{N}$, ${}^d\mathbf{M}\circ\mathbf{N}^d={}^d(\mathbf{M}\circ\mathbf{N})^d$;

(3) $(\mathbf{M}\circ_\omega\mathbf{N})^{\breve{}}=\mathbf{N}^{\breve{}}\circ_\omega\mathbf{M}^{\breve{}}$;

(4) Suppose $\mathcal{R}$ is a dual relation set on U, then $(\mathbf{M}\circ_\omega\mathbf{N})^d=\mathbf{M}\circ_\omega\mathbf{N}^d$, ${}^d(\mathbf{M}\circ_\omega\mathbf{N})={}^d\mathbf{M}\circ_\omega\mathbf{N}$, ${}^d\mathbf{M}\circ_\omega\mathbf{N}^d={}^d(\mathbf{M}\circ_\omega\mathbf{N})^d$.

Proposition 3.1. Let $\langle L,'\rangle$ be an orthocomplemented lattice with $|L|>4$ and let $U=L\backslash\{0,1\}$. Suppose **C** is a compatible contact relation on U other than the identity. Then for any four RCC11 relations **R**, **S**, **T**, **Q**, we have $\mathbf{R}\circ_\omega\mathbf{S}=\mathbf{T}\circ_\omega\mathbf{Q}$ provided that $\mathbf{R}\circ\mathbf{S}=\mathbf{T}\circ\mathbf{Q}$ holds, where $\circ_\omega$ is the weak RCC11 composition.

3.2 An approach for reducing the calculations of weak composition table

The above theorem suggests that, for a dual relation set $\mathcal{R}$, the work of constructing the weak composition table can be simplified drastically.

Suppose $\mathcal{R}$ is a dual relation set which is closed under inverse and contains $1'$. Let $\mathcal{S}$ be a dual generating set of $\mathcal{R}$ which is also closed under inverse. Denote $\mathcal{M}=\{\mathbf{R}\in\mathcal{S}:\mathbf{R}=\mathbf{R}^{\breve{}}\}$ and $\mathcal{R}\neq 1'$ and $\mathcal{N}=\{\mathbf{R}\in\mathcal{S}:\mathbf{R}\neq\mathbf{R}^{\breve{}}\}$. Write r,s,m,n to be the number of relations in $\mathcal{R},\mathcal{S},\mathcal{M},\mathcal{N}$ respectively. Then $s=m+n+1$ and $n=2k$ for some $k\in\mathcal{N}$.

To construct the weak **CT**, one should compute $\boldsymbol{M}\circ_w\boldsymbol{N}$ for each $\mathbf{M},\mathbf{N}\in\mathcal{R}$. Theorem 3.1 shows that the work can be simplified.

There are four cases, namely, (1) $\mathbf{M},\mathbf{N}\in\mathcal{S}$; (2) $\mathbf{M}\in\mathcal{S}$ and $\mathbf{N}\notin\mathcal{S}$; (3) $\mathbf{M}\notin\mathcal{S}$ and $\mathbf{N}\in\mathcal{S}$; (4) $\mathbf{M},\mathbf{N}\notin\mathcal{S}$.

For Case (2), since $\mathcal{S}$ is a dual generating set of $\mathcal{R}$, we can choose $\mathbf{R}\in\mathcal{S}$ such that $\mathbf{R}^d=\mathbf{N}$. Then $\mathbf{M}\circ_w\mathbf{N}=\mathbf{M}\circ_w\mathbf{R}^d=(\mathbf{M}\circ_\omega\mathbf{N})^d$ by (4) of Theorem 3.1. We reduce (2) to (1). Similarly, Case (3) and Case (4) can be reduced to (1). Therefore we only need to check Case (1). This can be further simplified. Suppose $\mathcal{M}=\{\mathbf{M}_1,\mathbf{M}_2,\ldots,\mathbf{M}_m\}$ and $\mathcal{N}=\{\mathbf{N}_1,\mathbf{N}_1^{\breve{}},\mathbf{N}_2,\mathbf{N}_2^{\breve{}},\ldots,\mathbf{N}_k,\mathbf{N}_k^{\breve{}}\}$.

- For $\mathbf{M},\mathbf{N}\in\mathcal{M}$, note $\mathbf{M}_i\circ_\omega\mathbf{M}_j=(\mathbf{M}_j\circ_\omega\mathbf{M}_i)^{\breve{}}$. The work needed in this case is $(m\times(m+1))/2$;
- For $\mathbf{M}\in\mathcal{M}$, $\mathbf{N}\in\mathcal{N}$ or $\mathbf{M}\in\mathcal{N}$, $\mathbf{N}\in\mathcal{M}$, note $\mathbf{M}_i\circ_\omega\mathbf{N}_j^{\breve{}}=(\mathbf{N}_j\circ\mathbf{M}_i)^{\breve{}}$ and $\mathbf{N}_j^{\breve{}}\circ_\omega\mathbf{M}_i=(\mathbf{M}_i\circ\mathbf{N}_j)^{\breve{}}$. The work needed in this case is $2m\times k$;
- For $\mathbf{M},\mathbf{N}\in\mathcal{N}$, note the following equations hold:

$$\mathbf{N}_i\circ_\omega\mathbf{N}_j=(\mathbf{N}_j^{\breve{}}\circ_\omega\mathbf{N}_i^{\breve{}})^{\breve{}},\mathbf{N}_i\circ_\omega\mathbf{N}_j^{\breve{}}=(\mathbf{N}_j\circ_\omega\mathbf{N}_i^{\breve{}})^{\breve{}},\mathbf{N}_i^{\breve{}}\circ_\omega\mathbf{N}_j=(\mathbf{N}_j^{\breve{}}\circ_\omega\mathbf{N}_i)^{\breve{}}.$$

The work needed in this case is $2k^2+k$.

Therefore the total work needed to construct the weak **CT** is $T=(m+n)(m+n+1)/2=s(s-1)/2$.

3.3 Dual relations of RCC systems

In this subsection we assume $\langle L,'\rangle$ is an orthocomplemented lattice with $|L|>4$ and let $U=L\backslash\{0,1\}$. We also suppose **C** is a compatible contact relation on U other than the identity.

Table 1 Dual operations on RCC7

R	PP	PP˘	PON	POD	DN	ECD	1′
$\mathbf{R}^d$	DN	POD	PON	PP˘	PP	1′	ECD
${}^d\mathbf{R}$	POD	DN	PON	PP	PP˘	1′	ECD
${}^d\mathbf{R}^d$	PP˘	PP	PON	DN	POD	ECD	1′

Table 2 Dual operations on RCC11

R	TPP	TPP˘	NTPP	NTPP˘	PON	PODY	PODZ	ECN	ECD	DC	1′
$\mathbf{R}^d$	ECN	PODY	DC	PODZ	PON	TPP˘	NTPP˘	TPP	1′	NTPP	ECD
${}^d\mathbf{R}$	PODY	ECN	PODZ	DC	PON	TPP	NTPP	TPP˘	1′	NTPP˘	ECD
${}^d\mathbf{R}^d$	TPP˘	TPP	NTPP˘	NTPP	PON	ECN	DC	PODY	ECD	PODZ	1′

Example 3.1. RCC5, RCC8 and RCC10 are not dual on L. Note that $\mathbf{PP}^d$ is not in RCC5, $\mathbf{TPP}^d$ is not in RCC8, and $\mathbf{POD}^d$ is not in RCC10. But by table 1 and table 2, RCC7 and RCC11 are clearly dual relation sets.

Moreover, for RCC7 and RCC11, we have $\mathcal{S}_7=\{1',\mathbf{PP},\mathbf{PP}^{\breve{}},\mathbf{PON}\}$ is a dual generating set of $\mathcal{R}_7$; and $\mathcal{S}_{11}=\{1',\mathbf{TPP},\mathbf{TPP}^{\breve{}},\mathbf{NTPP},\mathbf{NTPP}^{\breve{}},\mathbf{PON}\}$ is a dual generating set of $\mathcal{R}_{11}$.

By table 1 and table 2, $\mathcal{R}_7$ and $\mathcal{R}_{11}$ are closed under inverse and ${}^d\mathbf{R}^d=\mathbf{R}^{\breve{}}$ for $\mathbf{R}\in\mathcal{S}_7$ or $\mathbf{R}\in\mathcal{S}_{11}$. Moreover, for $\mathbf{M},\mathbf{N}\in\mathcal{S}_7$ or $\mathcal{R}_{11}$, by ${}^d\mathbf{M}\circ\mathbf{N}^d=({}^d\mathbf{M}^d)\circ({}^d\mathbf{N}^d)=\mathbf{N}^{\breve{}}\circ\mathbf{M}^{\breve{}}$, we have the following:

Proposition 3.2. For $\mathbf{M},\mathbf{N}\in\mathcal{S}_7$ or $\mathcal{R}_{11}$, we have ${}^d\mathbf{M}\circ\mathbf{N}^d=\mathbf{N}^{\breve{}}\circ\mathbf{M}^{\breve{}}$

By this proposition and Theorem 3.1, we have the following equations:

(1) $\mathbf{PODY}\circ\mathbf{PODY}=\mathbf{TPP}^{\breve{}}\circ\mathbf{TPP}$; (2) $\mathbf{PODY}\circ\mathbf{PODZ}=\mathbf{TPP}^{\breve{}}\circ\mathbf{NTPP}$;

(3) $\mathbf{PODY}\circ\mathbf{ECN}=\mathbf{TPP}^{\breve{}}\circ\mathbf{TPP}$; (4) $\mathbf{PODY}\circ\mathbf{DC}=\mathbf{TPP}^{\breve{}}\circ\mathbf{NTPP}^{\breve{}}$;

(5) $\mathbf{PODZ}\circ\mathbf{PODY}=\mathbf{NTPP}^{\breve{}}\circ\mathbf{TPP}$; (6) $\mathbf{PODZ}\circ\mathbf{PODZ}=\mathbf{NTPP}^{\breve{}}\circ\mathbf{NTPP}$;

(7) $\mathbf{PODZ}\circ\mathbf{ECN}=\mathbf{NTPP}^{\breve{}}\circ\mathbf{TPP}^{\breve{}}$; (8) $\mathbf{PODZ}\circ\mathbf{DC}=\mathbf{NTPP}^{\breve{}}\circ\mathbf{NTPP}^{\breve{}}$;

(9) $\mathbf{ECN}\circ\mathbf{PODY}=\mathbf{TPP}\circ\mathbf{TPP}$; (10) $\mathbf{ECN}\circ\mathbf{PODZ}=\mathbf{TPP}\circ\mathbf{NTPP}$;

(11) $\mathbf{ECN}\circ\mathbf{ECN}=\mathbf{TPP}\circ\mathbf{TPP}^{\breve{}}$; (12) $\mathbf{ECN}\circ\mathbf{DC}=\mathbf{TPP}\circ\mathbf{NTPP}^{\breve{}}$;

(13) **DC**∘**PODY**=**NTPP**∘**TPP**; (14) **DC**∘**PODZ**=**NTPP**∘**NTPP**;

(15) **DC**∘**ECN**=**NTPP**∘**TPP**$^{\sim}$; (16) **DC**∘**DC**=**NTPP**∘**NTPP**$^{\sim}$.

Note by Proposition 3.1, the relational composition ∘ in above equations can be replaced by weak composition $\circ_w$.

We now apply the approach described in Section 3.2 to RCC7 and RCC11. Set $t=T/n^2$ to be the ratio of the work needed in our approach to that using the cell-by-cell checking.

RCC7 $r=7$, $s=4$, $m=1$, $n=2$, $T=6$ and $t=6/49<1/8$;

RCC11 $r=11$, $s=6$, $m=1$, $n=4$, $T=15$ and $t=15/121<1/8$;

4 Complemented Closed Disk Algebra

This section shall provide a representation for the relation algebra determined by the RCC11 **CT**. In what follows, we write by $\tau_{11}:\mathcal{R}_{11}\times\mathcal{R}_{11}\to 2^{\mathcal{R}_{11}}$ the (abstract) RCC11 **CT** given in Ref.[8], which is also called a weak composition table there.

4.1 When is a composition triad extensional?

For an RCC model A, or more general, a contact structure $\langle L,\mathbf{C}\rangle$ on an orthocomplemented lattice, we say a composition triad $\langle\mathbf{R},\mathbf{T},\mathbf{S}\rangle$ in τ_{11} is extensional if $\mathbf{T}\subseteq\mathbf{R}\circ\mathbf{S}$. In Ref.[8], Düntsch has shown that in general the RCC11 **CT** is not extensional. As a matter of fact, he has determined for each cell $\langle\mathbf{R},\mathbf{S}\rangle$ whether or not $\mathbf{R}\circ_{\omega}\mathbf{S}=\mathbf{R}\circ\mathbf{S}$ is true for all RCC models. Our intention now is to give an exhaustive investigation of the extensionality of the RCC11 table. We want to indicate, for each triad $\langle\mathbf{R},\mathbf{S},\mathbf{T}\rangle$ with T an entry in the cell specified by the pair $\langle\mathbf{R},\mathbf{S}\rangle$, whether or not the following condition $\mathbf{T}(x,y)\to\exists z(\mathbf{R}(x,z)\wedge\mathbf{S}(z,y))$ holds for all RCC models.*

Table 3 RCC11 weak compositions should check

$\circ_\omega$	**TPP**	**TPP**$^{\sim}$	**NTPP**	**NTPP**$^{\sim}$	**PON**
TPP	?	?	?	?	?
TPP$^{\sim}$	?		?		?
NTPP	?		?	?	?
NTPP$^{\sim}$				?	?
PON					?

To make the calculations simple, we consider only strong RCC models, namely those models which satisfy the INT property. This cannot be too restrictive since stand RCC models of the Euclidean spaces are strong.

The following proposition suggests the approach specified in Section 3.2 can be used to reduce the calculations.

Proposition 4.1. Suppose A is an RCC model and **R**, **S**, **T** are three RCC11 relations on $U=A\backslash\{0,1\}$. Then the following conditions are equivalent:

(1) $\mathbf{T}\subseteq\mathbf{R}\circ_\omega\mathbf{S}$; (2) $\mathbf{T}^d\subseteq\mathbf{R}\circ_\omega\mathbf{S}^d$; (3) ${}^d\mathbf{T}\subseteq{}^d\mathbf{R}\circ_\omega\mathbf{S}$;

(4) ${}^d\mathbf{T}^d\subseteq{}^d\mathbf{R}\circ_\omega\mathbf{S}^d$; (5) $\mathbf{T}^{\sim}\subseteq\mathbf{S}^{\sim}\circ_\omega\mathbf{T}^{\sim}$.

Proof: The proofs are straightforward and leave to the reader.

Recall $\mathcal{S}_{11}$={1′,**TPP**,**TPP**$^{\sim}$,**NTPP**,**NTPP**$^{\sim}$,**PON**}. Let $\mathcal{M}_{11}$={**PON**}, $\mathcal{N}_{11}$={**TPP**,**TPP**$^{\sim}$,**NTPP**,**NTPP**$^{\sim}$}. Applying Proposition 4.1 and the approach described in Section 3.2, we need only to calculate the 15 weak compositions appeared in table 3. The results are given in table 4.

The verifications are similar to that given in Ref.[9] for RCC8 weak **CT**. Moreover, constructions given in Ref.[9, table 4, table 5] can also be applied for the RCC11 weak compositions. As a matter of fact, for any cell entry

* A similar and more detailed interpretation for RCC8 **CT** has been given in Ref.[9].

R in table 4 which is other than **PODY**, **PODZ**, **ECD**, we have: (1) if $a^{\times}$ is attached to **R**, the construction given in table 4 of Ref.[9] for corresponding RCC8 cell entry is still valid; (2) if this is not the case, entreating the counter-example constructed in table 5 of Ref.[9] will be enough. In particular, for strong RCC models, we have by table 3 of Ref.[9].

TPP∘**NTPP**=**NTPP**∘**TPP**=**NTPP**∘**NTPP**=**NTPP**;

TPP∘**TPP**=**TPP**∪**NTPP**;

NTPP∘**NTPP**$^{\smile}$=1′∘**TPP**∪**TPP**$^{\smile}$∪**NTPP**∪**PON**∪**ECN**∪**DC**;

NTPP$^{\smile}$∘**NTPP**=1′∪**TPP**$^{\smile}$∪**PON**∪**PODY**∪**PODZ**.

There are still 11 cell entries to be settled. For the two negative triads, ⟨**TPP**$^{\smile}$,**PODY**$^{\times}$,**TPP**⟩ and ⟨**TPP**$^{\smile}$,**PODY**$^{\times}$,**NTPP**⟩, take $p,q\in U$ with p**NTPP**q, set $q=q$, $c'=q-p$, then $a\wedge c=p$. Note by a**TPP**$^{\smile}c'$ we have a**PODY**c, but there cannot exist a region b with a**TPP**$^{\smile}b$ and $b\leq c$ since $a\wedge c=p$ is already a non-tangential proper part of a. For the rest positive composition triads, we can choose a region b with the desired property. These constructions are summarized in table 5.

Table 4 Reduced 'extensional' RCC11 **CT**, where **T**=**TPP**, **N**=**NTPP**, **Ti**=**TPP**$^{\smile}$, **Ni**=**NTPP**$^{\smile}$, **PN**=**PON**, **PDY**=**PODY**, **PDZ**=**PODZ**

∘$_\omega$	**T**	**Ti**	**N**	**Ni**	**PN**
T	**T**, **N**	1′, **T**, **Ti**, **DC**, **PN**$^{\times}$, **ECN**$^{\times}$	**N**	**Ti**$^{\times}$, **Ni**, **PN**$^{\times}$, **ECN**$^{\times}$, **DC**	**T**, **N**, **PN**, **ECN**, **DC**
Ti	1′, **T**, **Ti**, **PN**$^{\times}$, **PDY**$^{\times}$,**PDZ**		**T**$^{\times}$, **N**, **PN**$^{\times}$, **PDY**$^{\times}$, **PDZ**		**Ti**, **Ni**, **PN**, **PDY**, **PDZ**
N	**N**		**N**	1′, **T**, **Ti**, **N**, **Ni**, **PN**, **ECN**, **DC**	**T**, **N**, **PN**, **ECN**, **DC**
Ni			1′, **T**, **Ti**, **N**, **Ni**, **PN**, **PDY**, **PDZ**		**Ti**, **Ni**, **PN**, **PDY**, **PDZ**
PN					1′, **T**, **Ti**, **N**, **Ni**, **PN**, **DC**, **PDY**, **PDZ**, **ECN**, **ECD**

Table 5 Positive RCC11 weak compositions and instances of the region b

⟨**TPP**$^{\smile}$,**PODZ**,**TPP**⟩	Set $b=a\wedge c$
⟨**TPP**$^{\smile}$,**PODZ**,**NTPP**⟩	Take m with c'**NTPP**m**NTPP**a, set $b=a-m$
⟨**TPP**$^{\smile}$,**PODY**,**PON**⟩	Take $m=c'$, n**NTPP**$(a\wedge c)$, set $b=m+n$
⟨**TPP**$^{\smile}$,**PODZ**,**PON**⟩	Take m**NTPP**c', $n=a\wedge c$, set $b=m+n$
⟨**NTPP**$^{\smile}$,**PODY**,**PON**⟩	Take m**NTPP**c', n**NTPP**$(a\wedge c)$, set $b=m+n$
⟨**NTPP**$^{\smile}$,**PODZ**,**PON**⟩	Take m**NTPP**c', n**NTPP**$(a\wedge c)$, set $b=m+n$
⟨**PON**,**PODY**,**PON**⟩	Take m**NTPP**c', n**NTPP**a', set $b=m+n$
⟨**PON**,**PODZ**,**PON**⟩	Take m**NTPP**c', n**NTPP**a', set $b=m+n$
⟨**PON**,**ECD**,**PON**⟩	Take m**NTPP**c', n**NTPP**a', set $b=m+n$

4.2 Topological characterization of RCC11 relations in $\mathcal{L}$

Recall RC(R^2), the standard RCC model associated to the Euclidean plane, contains all regular closed subsets of R^2, and two (nonempty) regions are said to be connected provided that they have nonempty intersection.

Our domain of regions, denoted by D, is a sub-domain of RC(R^2) and contains two classes of regions: the closed disks and their complements in RC(R^2). We denoted by D_1 the class of closed disks, by D_2 the class of their complements and call for convenience regions in D_2 complement disks. Define a binary relation **C** on D as follows: for two regions $a,b\in D$, a**C**b if $a\cap b\neq\varnothing$. Clearly this relation is a contact relation on U. In contrast with the closed disk algebra for RCC8 table given in Ref.[8,13], we call the contact relation algebra on this domain the complemented closed disk algebra, written $\mathcal{L}$. In what follows we shall show this CRA is finite and contains RCC11 as its atoms, and it is indeed a representation of the relation algebra determined by the RCC11 **CT**.

Write $L=D\cup\{\varnothing,R^2\}$. Then L with the usual inclusion ordering is an orthocomplemented lattice. Based on the contact relation **C** on D, we can define RCC11 relations on D (see Section 2 of this paper).

The following theorem gives a topological characterization of these relations:

Theorem 4.1. The RCC11 relations on D have the following characterization:

(1) $x1'y$ iff $x=y$;

(2) x**TPP**y iff $x\subseteq y$, $x\neq y$ and $\partial x\cap\partial y\neq\varnothing$;

(3) x**TPP**$^{\breve{}}y$ iff $x\supseteq y$, $x\neq y$ and $\partial x\cap\partial y\neq\varnothing$;

(4) x**NTPP**y iff $x\subseteq y$, $x\neq y$ and $\partial x\cap\partial y\neq\varnothing$;

(5) x**NTPP**$^{\breve{}}y$ iff $x\supseteq y$, $x\neq y$ and $\partial x\cap\partial y\neq\varnothing$;

(6) x**PON**y iff $x^\circ\cap y^\circ\neq\varnothing$, $x\not\subseteq y, y\not\subseteq x$, and $x\cup y\neq R^2$;

(7) x**PODY**y iff $x^\circ\cap y^\circ\neq\varnothing$, $\partial x\cap\partial y\neq\varnothing$ and $x\cup y\neq R^2$;

(8) x**PODY**y iff $x^\circ\cap y^\circ\neq\varnothing$, $\partial x\cap\partial y\neq\varnothing$ and $x\cup y\neq R^2$;

(9) x**ECN**y iff $x^\circ\cap y^\circ\neq\varnothing$, $x\cap y\neq\varnothing$ and $x\cup y\neq R^2$;

(10) x**ECD**y iff $x^\circ\cap y^\circ\neq\varnothing$, $x\cap y\neq\varnothing$ and $x\cup y\neq R^2$;

(11) x**DC**y iff $x\cap y\neq\varnothing$.

Proof: The proofs are routine and leave to the reader.

From this theorem we know that these relations on D are precisely the restrictions of the corresponding RCC11 relations in RC(R^2) to D.

4.3 The composition of the complemented closed disk algebra

Now we shall show that the composition operation of $\mathcal{L}$ is precisely that one specified by the RCC11 **CT**. What we should do is to indicate, for each triad $\langle$**R**,**T**,**S**$\rangle$ with **T** an entry in the cell specified by the pair $\langle$**R**,**S**$\rangle$, whether or not the following condition hold: $\mathbf{T}(x,y)\rightarrow(\exists z\in D)(\mathbf{R}(x,z)\wedge\mathbf{S}(z,y))$.

Note the approach described in Section 3.2 is also valid for the present purpose. This is due to the facts that (i) RCC11 relation on D is a dual relation set which contains $1'$ and is closed under inverse; (ii) $\mathcal{S}_{11}$={$1'$,**TPP**,**TPP**$^{\breve{}}$,**NTPP**, **NTPP**$^{\breve{}}$,**PON**} is a dual generating set which is also closed under inverse; (iii) Proposition 4.1 is still valid for $\mathcal{L}$. As a result, we need only to calculate the 15 compositions appeared in table 3.

To begin with, we first show the **NTPP** relation on D satisfies the interpolation property.

Lemma 4.1. Given any two regions a, c in D with a**NTPP**c, there exists another region $b\in D$ with a**NTPP**b**NTPP**c.

Proof: By the topological characterization of the **NTPP** relation given in Theorem 4.1, we know that a**NTPP**c if and only if $a\subset c^\circ$. There are three cases:

Case I: a,c are closed disks. In this case, ∂a and ∂c are two non-tangential circles and ∂a is inside ∂c. Then we can find another circle B between these two circles. Taking b as the closed disk bounded by B, then b satisfies the desired property.

Case II: a,c are complement disks. In this case, ∂a and ∂c are two non-tangential circles and ∂c is inside ∂a. Then we can find another circle B between these two circles. Taking b as the complement disk bounded by B, then b satisfies the desired property.

Case III: a is a closed disk and c is a complement disk, ∂a and ∂c are two separated circles and the distance between them is non-zero. Then we can find another circle B such that ∂a is inside B and B is separated from ∂c. Taking b as the closed disk bounded by B, then b satisfies the desired property.

Proposition 4.2. In the complemented closed disk algebra $\mathcal{L}$ the following equations **NTPP**$\circ$**NTPP**=**NTPP**, **TPP**$\circ$**NTPP**= **NTPP** and **NTPP**$\circ$**TPP**=**NTPP** hold.

Proof: Note the "$\subseteq$" part of these equations follows directly from the definitions and the first equation is then clear by above lemma.

For the second equation, suppose a**NTPP**c in D, we want to find b such that a**TPP**b**NTPP**c. There are three cases:

Case I: a,c are closed disks. In this case, ∂a and ∂c are two non-tangential circles and ∂a is inside ∂c. Then we can find another circle B such that ∂a is internally tangent to B and B is inside the circle ∂c. Taking b as the closed disk bounded by B, then b satisfies the desired property.

Case II: a,c are complement disks. In this case, ∂a and ∂c are two non-tangential circles and ∂c is inside ∂a. Then we can find another circle B such that B is internally tangent to ∂a and ∂c is inside B. Taking b as the closed disk bounded by B, then b satisfies the desired property.

Case III: a is a closed disk and c is a complement disk, ∂a and ∂c are two separated circles and the distance between them is non-zero. Then we can find another circle B such that ∂a is internally tangent to B and B is separated from ∂c. Taking b as the closed disk bounded by B, then b satisfies the desired property.

The proof of the last equation is similar.

The following proposition proves the remainder 12 equations in CCA.

Proposition 4.3. In the complemented closed disk algebra $\mathcal{L}$, the following composition equations hold.

(C-1) **TPP**∘**TPP**=**TPP**∪**NTPP**;

(C-2) **TPP**∘**TPP**˘=1′∪**TPP**∪**TPP**˘∪**PON**∪**ECN**∪**DC**;

(C-3) **TPP**∘**NTPP**˘=**TPP**˘∪**NTPP**˘∪**PON**∪**ECN**∪**DC**;

(C-4) **TPP**∘**PON**=**TPP**∪**NTPP**∪**PON**∪**ECN**∪**DC**;

(C-5) **TPP**˘∘**TPP**=1′∪**TPP**∪**TPP**˘∪**PON**∪**PODY**∪**PODZ**;

(C-6) **TPP**˘∘**NTPP**=**TPP**∪**NTPP**∪**PON**∪**PODY**∪**PODZ**;

(C-7) **TPP**˘∘**PON**=**TPP**˘∪**NTPP**˘∪**PON**∪**PODY**∪**PODZ**;

(C-8) **NTPP**∘**NTPP**˘=1′∪**TPP**∪**TPP**˘∪**NTPP**∪**NTPP**˘∪**PON**∪**ECN**∪**DC**;

(C-9) **NTPP**∘**PON**=**TPP**∪**NTPP**∪**PON**∪**ECN**∪**DC**;

(C-10) **NTPP**˘∘**NTPP**=1′∪**TPP**∪**TPP**˘∪**NTPP**∪**NTPP**˘∪**PON**∪**PODY**∪**PODZ**;

(C-11) **NTPP**˘∘**PON**=**TPP**˘∪**NTPP**˘∪**PON**∪**PODY**∪**PODZ**;

(C-12) **PON**∘**PON**=1′∪**TPP**∪**TPP**˘∪**NTPP**∪**NTPP**˘∪**PON**∪**PODY**∪**PODZ**∪**ECN**∪**ECD**∪**DC**.

Proof: Since regions in D are either closed disks or the complement of closed disks, the above equations can be verified using elementary theory for circles (such as, internally tangent, externally tangent, containment, disjoint, etc.).

As a result, we know that the complemented closed disk algebra has 11 atoms and its composition is just as the one given in the RCC11 **CT**.

Theorem 4.2. The relation algebra determined by the RCC11 **CT** can be represented by the complemented closed disk algebra.

5 Summary and Outlook

This paper explores several important relation-algebraic questions arising in the RCC11 theory. For the RCC11 table, we have shown in Section 4 of this paper the complemented closed disk algebra, whose domain contains only the closed disks and closures of their complements in the real plane, is an extensional model.

Future work will investigate the contact relation algebra of various small domains of regions which admits more operations than complementation, e.g., finite unions or finite intersections. In particular, the (complemented) Worboys-Bofakos model[20] deserves a detailed study with the tools of relation algebra. Note that the 9-intersection principle can be applied to these domains, we can compare the expressivity of RA logic with that of the 9-intersection model.

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