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Research on Multiobjective Optimization Based on Ecological Cooperation*

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Abstract: A multiobjective optimization using genetic algorithm based on ecological cooperation (ECGA) is proposed in this paper after analyzing the present studies. In the algorithm, an ecological population density competition equation is used for reference to describe the relation between multiple objectives and to direct the adjustment over the relation at individual and population levels. Simulation experiments prove that the algorithm has a better performance in finding the Pareto solutions.

Key words: multiobjective optimization; pareto solution; genetic algorithm; ecological cooperation; ecological population density competition equation

The solution of a real problem involved in multiobjective optimization (MO) must satisfy all optimization objectives simultaneously, and in general the solution is a set of indeterminacy points. The task of MO is to estimate the distribution of this solution set, then to find the satisfying solutions in it.[1,2]

Many methods of solving MO have been proposed in the literature. These methods generally can be classified into two categories. (Ⅰ) Traditional methods, including program-planning, pre-weighted and restraint increasing. The basic idea of these methods is converting multiple objectives into a single overall objective in one step or more, then computing and adjusting the ratio of each objective in the overall objective, until achieving the satisfactory solutions. However, in real problem these objectives cannot be combined into a single function because they are incommensurable. (Ⅱ) Multiobjective optimization using genetic algorithm. Since the study of VEGA[3] by Schaffer, several methods have been proposed for solving MO by GA. So far, the most successful approach to MO by GA seems to be based on Pareto ranking.[2,4] The Pareto-based ranking is a method of evaluating the individual by its degree of Pareto optimality in the current population and implements some cooperation of different objectives' individuals. While the Pareto-based ranking makes it possible to find Pareto optimal solutions by GA, a good sampling of the solutions from the Pareto optimal set is not guaranteed only by this technique.[5] That is, the population converges to small number of solutions due to the random genetic drift. To avoid this phenomenon, some work ex-

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ploited niche technique[6,7].

In this paper, the authors propose a novel method based on ecological cooperation for solving MO. To adjust automatically the relation of multiple objectives at individual and population levels, an explicit mathematic model should be exploited for guidance. Using ecological population density competition equation for reference, we describe the complicated, nonlinear relation of multiple objectives and adjust the relation at individual and population level simultaneously.

1 Definition of Multiobjective Optimization

The multiobjective optimization problem (MOP) is formulated as follows[1]:

$$\min_{x \in F} f_1(x), f_2(x), \ldots, f_M(x)$$

where $f_1(x), f_2(x), \ldots, f_M(x)$ are the objective functions to be minimized simultaneously, x is the decision variable and F is the feasible region. In this paper, we discuss no restraint MOP.

Definition 1. A point $x=(x_1, x_2, \ldots, x_n)$ is said to dominate another point $y=(y_1, y_2, \ldots, y_n)$ if $\forall\ x_i \leqslant y_i$ and $\exists\ i_0 / x_{i_0} \leqslant y_{i_0}$.

Definition 2. Given two feasible solutions x and y, if $\forall\ m=1,2,\ldots,M, f_m(x) \leqslant f_m(y)$ and $\exists\ m \mid f_m(x) < f_m(y)$, we say that x dominates y.

Definition 3. If a feasible solution x^* is not dominated by $\forall\ x \in F$, we call x^* a Pareto optimal solution.

x^* is a rational solution of MO. In practice a solution usually comprises large components and small components, so there is no rational optimal solution. We also call the set consisting of all the Pareto optimal solutions the Pareto optimality set. The first goal of solving MO is to obtain the Pareto optimality set, or to sample solutions from it.

2 Ecological Model for Multiobjective Optimization

In collaborative evolution individuals' self-status, living environment and competition with other individuals affect individuals' self-evolution. It's similar to the evolution from the point of view of ecology. Theoretical ecology has gone into it. Population in certain living environment is affected not only by itself's fitness but also the living environment and the collaborative competition with other populations. The last two factors can be implemented by population density at population level.

Lotka-Volterra competition equation as the theoretical model of population competition is introduced to describe populations' cooperation, based on which we proposed a collaborative evolutionary algorithm for MO in next section.

Given two populations N_1, N_2, the cooperation between them can be formulated as follows:

$$\frac{dN_1}{dt} = r_1 N_1 \left(\frac{K_1 - N_1 - a_{12} N_2}{K_1} \right) \quad (1)$$

$$\frac{dN_2}{dt} = r_2 N_2 \left(\frac{K_2 - N_2 - a_{21} N_1}{K_2} \right) \quad (2)$$

where K_1, K_2 are the living environment loads of populations N_1, N_2 without competition to each other; r_1, r_2 are individual's maximum increasing rates. a_{12}, a_{21} are competition coefficients. a_{ij} represents the suppression effect of individuals of population N_i from the individuals of population N_j.[8]

This model based on population density completely describes the main collaborative relations. Examining Eq. (1) and Eq. (2), an individual in population N_1 has a suppression effect on the increasing of itself's population, and the suppression effect's value is $1/K_1$. Population N_2 is the same as population N_1, but the value is $1/K_2$. Population N_1 has the suppression effect on individuals of population N_2, and the suppression effect's value is a_{21}/K_2.

Symmetrically, there is a value of a_{12}/K_1. The collaborative results rest on the relations of K_1, K_2, a_{12} and a_{21}.

In Eq. (1) and Eq. (2), let $dN_1/dt=0$, $dN_2/dt=0$, then we can draw the isolines of every population. The area under isoline is the population density increasing area, and the upper is decreasing area, as shown in Fig. 1.

(1) When $K_2/a_{21}<K_1$, $K_1/a_{12}>K_2$, isoline of population N_1 lies in the top of isoline of population N_2. Population N_1 always wins. Stable balance is reached when $N_1=K_1$ and $N_2=0$ (Fig. 1(a)).

(2) When $K_2/a_{21}>K_1$, $K_1/a_{12}<K_2$, isoline of population N_2, lie in the top of isoline of population N_1. Population N_2 will always win. Stable balance is reached when $N_2=K_2$ and $N_2=0$ (Fig. 1(b)).

(3) When $K_2/a_{21}<K_1$, $K_1/a_{12}<K_2$, anyone of populations N_1, N_2 could win, and suppress the opponent. There exist three balances. The balance at cross point is not stable, and the conditions of stable balance is $N_1=K_1$ and $N_2=0$ or $N_2=K_2$ and $N_2=0$. The winner depends on the ratio of the two populations' initial quantities (Fig. 1(c)).

(4) When $K_2/a_{21}>K_1$, $K_1/a_{12}>K_2$, these two populations cannot suppress the opponent, and there is only one balance point (cross point). The two populations can coexist in certain population density and under its living environment's loads.

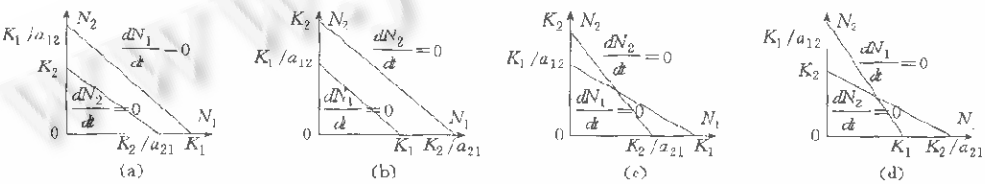


Fig. 1 Isolines of two collaborative populations

In a community comprising n different populations, competition equation can be formulated as follows:

$$\frac{dN_i}{dt}=r_iN_i\left\{\frac{K_i-N_i-\left(\sum_{j=1}^{n}a_{ij}N_j\right)}{K_i}\right\} \tag{3}$$

It's the cooperation model based on ecological population density. We could exploit this model to describe the relation of multiple objectives. Because the relation of multiple objectives is just collaborative coexistence, and it finally is stable, two restraints must be followed:

(1) The cooperation is based on population density, and all objectives must be converted into corresponding population density (or population scale). In this paper, if the scale of population N_i is increasing, the corresponding objective f_i will increase its ratio in the overall objective, which is defined as: $\overline{f_i}=\alpha N_i$, where α is a ratio constant, $\overline{f_i}$ is the average fitness of this population corresponding to the ith objective.

(2) Explicitly specify the values of K_i, K_j, a_{ij} and a_{ji} to guarantee that there is only collaborative coexistence among multiple objectives. That is, given two populations, the problem $K_1/a_{12}>K_2$, $K_2/a_{21}>K_1$ must be satisfied. To resolve it, we decompose it into two sub-problems:

(I) a_{12} and a_{21} must be less than 1 if $K_1=K_2$;

(II) $a_{12}<K_1/K_2<1/a_{21}$ if $K_1\neq K_2$.

3 An MO Algorithm Based on Ecological Cooperation (ECGA)

Exploiting Eq. (3), we proposed an MO algorithm based on ecological cooperation. Fig. 2 shows the algorithm (ECGA). The basic idea of ECGA is as follows:

(I) each objective corresponds to a population,

(II) in one iterative step, evolution process and cooperation process must be executed. The evolutionary pro-

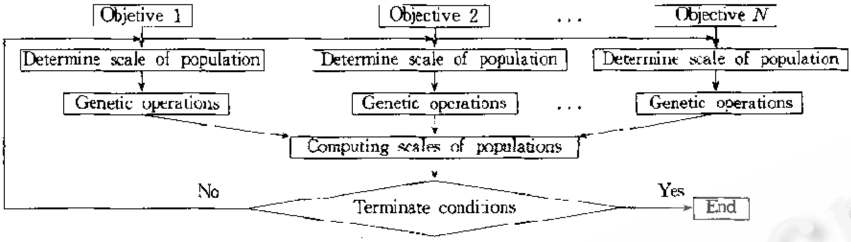


Fig. 2 Multiobjective optimization algorithm based on ecological cooperation

cess adopts GA's genetic operations, while the cooperation process adopts Eq. (3) to compute population density and to adjust the scales of populations. The scale of population is formulated as

$$N_i(t+1)=N_i(t)+\frac{dN_i}{dt}$$

(1) If the increasing of population N_i is positive, randomly generated dN_i/dt individuals join population N_i for enlarging the scale of N_i.

(2) If the increasing of population N_i is negative, according to the fitness of population N_i, dN_i/dt individuals with minimal fitness are deleted. The scale of population is reduced.

As a complete unit, ECGA pseudocode description is given here:

Step 1: for all objective functions $f_i(x)$

  Initialize a random population N_i corresponding to $f_i(x)$

Step 2: while (terminative condition is NOT satisfied)

  for all populations N_i

    General genetic operations are performed

    Computing dN_i/dt using Eq. (3)

    Determine next generation's scale of population N_i using equation:

$$N_i(t+1)=N_i(t)+dN_i/dt$$

  end for

 end while

Examining the adjusting strategies, the randomly generated individuals enlarge the scale of population (population density), which maintains the diversity of population and to a certain extent improves the global distribution of feasible solutions in the whole solution space. One or more individuals that have minimal fitness will be deleted. It's in agreement with natural selection doctrine, and will improve the fitness of this population in next iterative step. So this population could hold more competition strength, and its corresponding objective will obtain larger ratio in the overall objective. And the MO problem will achieve satisfying solutions. The algorithm (ECGA) using above iterative adjustment makes every population collaborative evolution.

In ECGA, not only multiple objectives' cooperation is realized, evolutionary individuals' self-suppress cooperation in a single objective is also realized. Eq. (1) and Eq. (2) can be rewritten as:

$$\frac{dN_1}{dt}=r_1N_1-\frac{r_1N_1^2}{K_1}-\frac{r_1N_1a_{12}N_2}{K_2}$$

$$\frac{dN_2}{dt}=r_2N_2-\frac{r_2N_2^2}{K_2}-\frac{r_2N_2a_{21}N_1}{K_2}$$

where the first item on the right side represents population increasing without density restraints. The 2nd and 3rd items respectively represent self-restrain in population and collaborative competition between populations.

In addition, we discuss the associative performance parameters: K_i, r_i, and a_{ij}. K_i is the environment load that depends on the given objective. The value of r_i is linearly decreasing along with the decreasing of N_i when $N_i=K_i$

and r_i is 0. So let $r_i=(K_i-N_i)/k_i$, where k_i is a coefficient. As a result, the optimal performance will lies on a_{ij}. Certainly the restraints (2) mentioned in last section should be satisfied. In Fig. 1(d), if restraints (2) is satisfied, the values of K_i, r_i, a_{ij} and a_{ji} will determine the convergence area of ECGA.

4 Experiment

We tested the performance of ECGA on three problems of different difficulties, and compared it with the niched Pareto GA (nPGA). The objective functions that define each one of these problems are listed in Table 1. In Table 2, the values of parameters for nPGA and ECGA are the same, so that it is easy to compare.

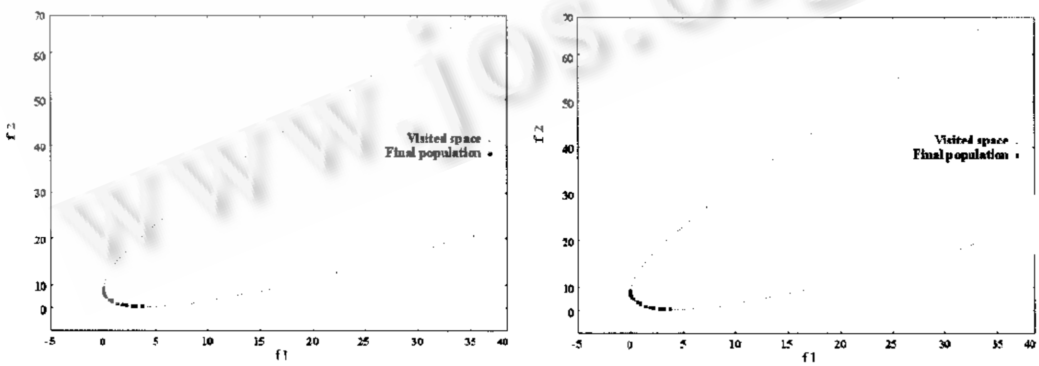
Table 1 Three MO problems

Problem	Objectives	Ranges
1	$\text{Min} f_1(x)=x^2$ $\text{Min} f_2(x)=(x-2)^2$	$x\in[-6,6]$
2	$\min_{x,y} f_1(x,y)=1/(x^2+y^2+1)$ $\min_{x,y} f_2(x,y)=(x^2+3y^2+1$	$y\in[-3,3]$
3	$\min_{x,y} f_1(x,y)=x+y+1$ $\min_{x,y} f_2(x,y)=(x^2+2y^2-1$	$y\in[-3,3]$

Table 2 The values of parameters

		Chromosome length	Population scale	Crossover probability	Mutation probability
Problem 1	nPGA	14	30	0.033	0.85
	ECGA	14	30	0.033	0.085
Problem 2	nPGA	40	60	0.033	0.70
	ECGA	40	69	0.033	0.70
Problem 3	nPGA	40	60	0.033	0.85
	ECGA	20	60	0.033	0.85

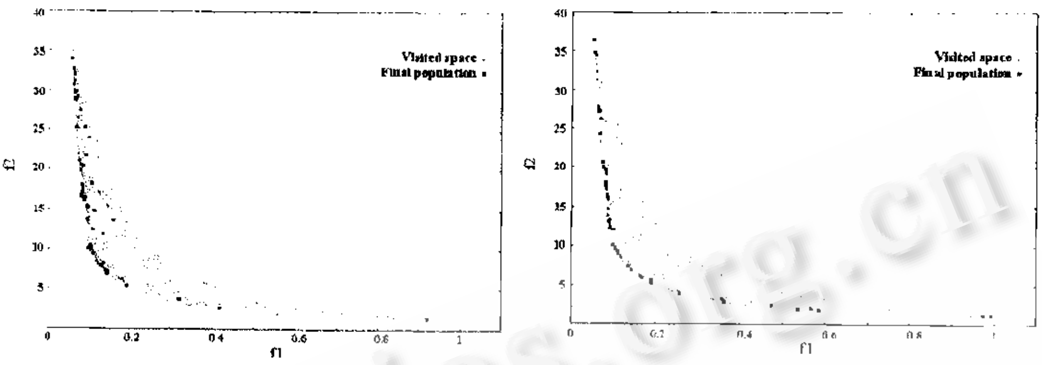
Problem 1 is fairly simple where there is only one variable x. In Fig. 3, visited space and final population are shown according to different algorithms. Notice that some individual of final population of nPGA is far away from the Pareto optimal set. This contrasts with our ECGA in which all individuals lie close to the Pareto optimal set.



(1) nPGA (2) ECGA

Fig. 3 Visited space and final population of problem 1

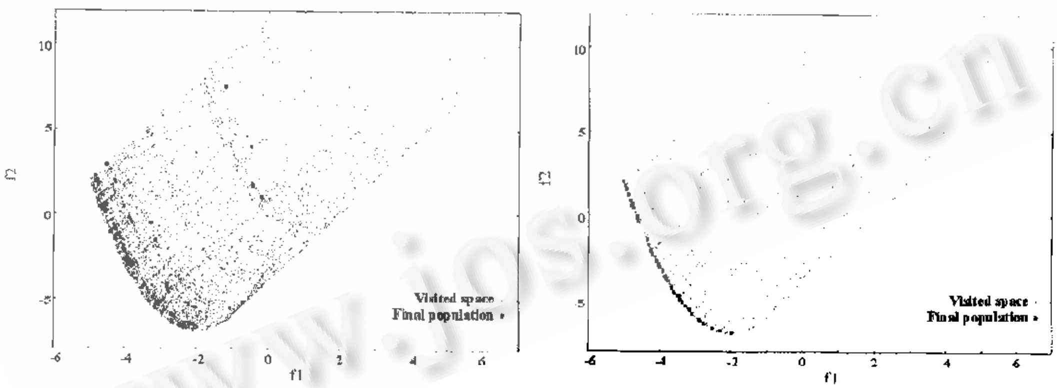
Problem 2 is slightly more difficult than problem 1. It has two variables, x and y, and in the objective function plane, the set of valid points is an area. Fig. 4 shows visited space and final population of the nPGA and our ECGA. Notice that the final population of our ECGA is more uniform.



(1) nPGA (2) ECGA

Fig. 4 Visited space and final population of problem 2

Finally, problem 3 is more difficult than problems 1 and 2, the valid region of the objective function plane is larger, and a more complex relation exists between the independent variables, x and y, and functions f_1 and f_2. Fig. 5 shows the visited space and the final population of the nPGA and our ECGA. There is a noticeable difference in performance between algorithms. Our ECGA visits almost one tenth of the space as nPGA does, but the final population of the ECGA achieves Pareto optimal set uniformly, whereas the nPGA appears to visit all the search spaces with a dissatisfying result.

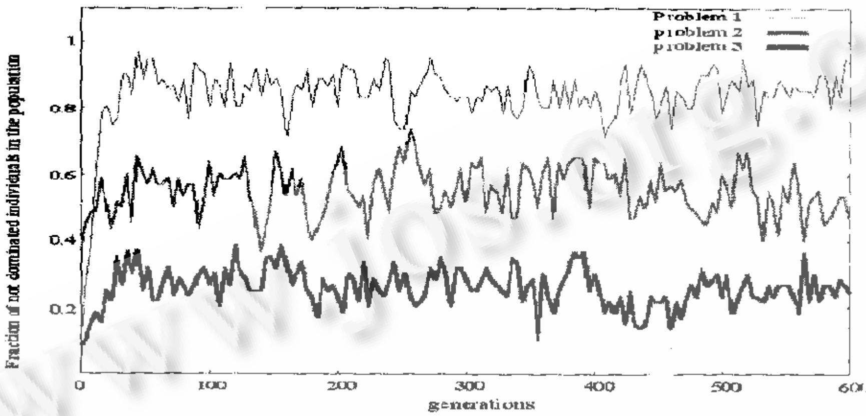


(1) nPGA (2) ECGA

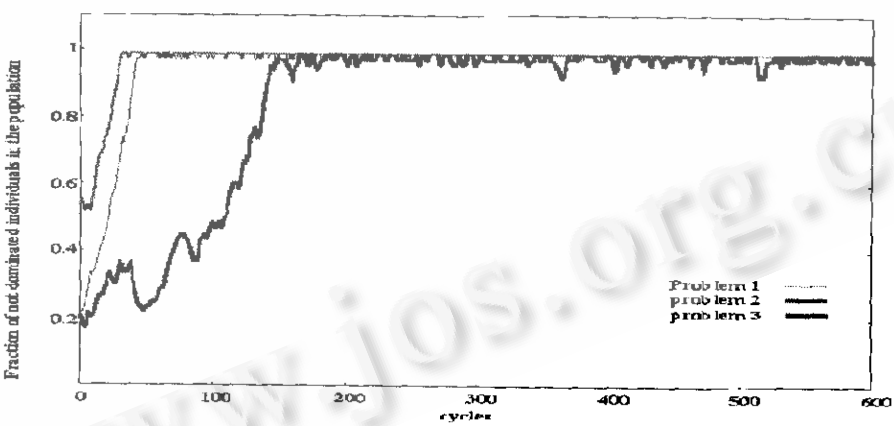
Fig. 5 Visited space and final population of problem 3

In conclusion, our ECGA searches smaller visited space and fewer points not belonging to Pareto optimal set are included by final population when we solve the three MO problems. It seems that the nPGA has visited the whole search space without a stronger concentration near the Pareto optimal set, and its final population contains many dominated points.

In Fig. 6, the ratio of non dominated individuals in final populations is compared among the three problems. Fig. 6 (1) shows the fraction of non-dominated individuals in the population of nPGA does not have stable behavior. As the problem difficulty increases, the value of the plot is reduced, indicating that the algorithm delivers a solution of decreasing quality as the difficulty of the problem grows. In our ECGA the plot is an almost monotonically increasing curve. As the problem difficulty increases, the fraction of non-dominated individuals takes more evaluations to reach the value of 1.0, but once it reaches this value it stays very close to it. This behavior indicates that the Pareto optimal solutions of our ECGA have stronger concentration ability (Fig. 6(2)).



(1)



(2)

Fig. 6 The ratio of non-dominated individuals in population

5 Conclusion

The interest in multiobjective optimization lies in that most real problems involve more than one optimization objective. The bottle-neck of MO is to describe the complex relations of multiple objectives. In this paper, we proposed an MO algorithm (ECGA) based on ecological cooperation, using dynamical equation of population competition at ecology to describe the complex, nonlinear relations of multiple objectives and to adjust the relation on individual and population levels simultaneously. The advantages of our ECGA are guaranteeing the uniform distribution of solutions and strengthening the concentration ability of solutions.

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基于生态协同的多目标优化研究

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摘要:在分析现有多目标优化技术的基础上,提出了一种基于生态协同的多目标优化算法.此算法借鉴生态学中的生态种群密度竞争方程来描述多目标间的复杂关系,可以同时从个体和种群层次指导多目标之间关联程度的调整.实验结果表明,此算法更易于寻找多目标优化问题的满意解.

关键词:多目标优化;非劣解;遗传算法;生态协同;生态种群密度竞争方程

中图法分类号:TP18 **文献标识码**:A