1000-9825/2001/12(03)0340-07 ©2001 Journal of Software 软件学报 Vol. 12, No. 3

A Polynomial Time Judgement for Liveness of Asymmetric Choice Nets*

JIAO Li, LU Wei-ming

(Institute of Mathematics, Academy of Mathematics and System Science, The Chinese Academy of Sciences, Beijing 100080, China)

E-mail: ljiao@math20.math.ac.cn

Received December 8, 1999; accepted April 25, 2000

Abstract: The problem on liveness judgement is still open in Petri nets. The paper studies liveness on Asymmetric Choice nets (AC nets) by structure analysis theory. Firstly, some known results on liveness are discussed. Then a sufficient condition by S-invariant for the liveness of AC nets is presented and the corresponding polynomial-time algorithm is got. Finally a simple necessary-and-sufficient condition on monotonicity of bounded AC nets is presented.

Key words: AC net; liveness; liveness monotonicity; polynomial time

Petri nets are well known models for the representation and analysis of distributed systems[1]. Liveness is one of the main behavioral properties of Petri nets. For classes of P/T nets with a restricted modeling power, liveness can be structurally characterized and efficiently decided under the boundedness hypothesis[2~8].

A common property to these restricted classes is that liveness is ensured by checking some particular sets of places. Deadlocks are critical system parts for liveness analysis, because transitions may never be enabled again if they contain places of an unmarked deadlock in their preset. Like deadlocks, there are also system parts which will never lose all tokens again after they have once been marked. For these classes, the existence of deadlocks keeping marked is a necessary and sufficient condition for the net to be live. Hence every deadlock of the net must be controlled to ensure that the deadlock remains marked for every reachable state. There are two ways of controlling a deadlock. The first one relies on the concept of trap[9]. The second one is based on the concept of invariant[4].

For asymmetric choice nets (AC nets), a lot of research efforts on liveness analysis still go on and some good results have been got[10~14].

In this paper, we get a sufficient condition for liveness of AC nets by controlled deadlocks. Controlled deadlocks cannot get unmarked. The mechanism that prevents a controlled deadlock from getting unmarked is quite different from a marked trap inside the deadlock. This condition decreases the complexity to check liveness for some AC nets. On the basis of Refs. [10~13], we also present and prove a necessary and sufficient condition on liveness monotonicity of bounded AC nets in order to simplifying the previous judgement method.

The paper is organized as follows. In Section 1, we introduce some basic concepts and notations of P/T nets

---

* This project is supported by the National Natural Science Foundation of China under Grant No. 69773016 (国家自然科学基金) and the National Key Fundamental Research Program of China under Grant No. G1998030416 (国家重点基础研究专项经费).

JIAO Li was born in 1964. She is a Ph.D. student at the Institute of Mathematics, Academy of Mathematics and System Sciences, the Chinese Academy of Sciences. Her research interests are Petri nets and algorithm. **LU Wei-ming** was born in 1941. He is a professor and doctoral supervisor of the Institute of Mathematics, Academy of Mathematics and System Science, the Chinese Academy of Sciences. His current research areas include Petri nets, algorithm and software technology.

used here. Besides several important properties and results of AC nets, we give our results and prove them in Section 2. Finally, we discuss future work and conclude the paper.

1 Basic Definitions and Notations

1.1 Place/Transition net system

Definition 1.1. A net is a tuple $N=(P,T;F)$ with

(a) P and T are finite and disjoint sets, which are places and transitions, respectively;

(b) $F\subseteq(P\times T)\cup(T\times P)$;

(c) $\mathrm{dom}(F)\cup\mathrm{cod}(F)=P\cup T$, where $\mathrm{dom}(F)=\{x\mid\exists\ y:(y,x)\in F\}$, $\mathrm{cod}(F)=\{x\mid\exists\ y:(y,x)\in F\}$.

The preset of a node $x\in P\cup T$ is defined as ${}^{\bullet}x=\{y\in P\cup T\mid(y,x)\in F\}$. The postset of a node $x\in P\cup T$ is defined as $x^{\bullet}=\{y\in P\cup T\mid(x,y)\in F\}$. The preset (postset) of a set $X\subseteq(P\cup T)$ is the union of the preset (postset) of the elements of X.

Definition 1.2. Let $N=(P,T;F)$ be a net.

1. A marking of a net $N=(P,T;F)$ is a mapping $M:P\to\mathcal{N}$, where $\mathcal{N}=\{0,1,2,3,\ldots\}$.
2. The pair (N,M_0) is a Place/Transition net system or marked net, where M_0 is the initial marking.
3. A transition $t\in T$ is enabled under M, written as $M[t>$, iff $\forall\ p\in{}^{\bullet}t:M(p)>0$.
4. If $M[t>$, the transition t may occur, resulting in a new marking M', written as $M[t>M'$ with

$$M'(p)=\begin{cases}M(p)-1 & \text{if } p\in{}^{\bullet}t\backslash t^{\bullet}\\ M(p)+1 & \text{if } p\in t^{\bullet}\backslash{}^{\bullet}t\\ M(p) & \text{otherwise}\end{cases}$$

for all $p\in P$.

5. The set of all reachable markings, written as $[M_0>$, of a marking M_0 is the smallest set, such that $M_0\in[M_0>$ and $M\in[M_0>\wedge M[t>M'\Rightarrow M'\in[M_0>$ hold.
6. If $M_0[t_1>M_1[t_2>\ldots[t_n>M_n$, then $\sigma=t_1t_2\ldots t_n$ is an occurrence sequence.

Definition 1.3. Let (N,M_0) be a net system and $N=(P,T;F)$.

1. A transition $t\in T$ is live under M_0, iff $\forall\ M\in[M_0>,\exists\ M'\in[M>:M'[t>$.
2. The net N is dead under M_0, iff there does not exist $t\in T$ such that $M_0[t>$.
3. The net N is deadlock-free under M_0, iff $\forall\ M\in[M_0>,\exists\ t\in T:M[t>$.
4. The net N is live under M_0, iff $\forall\ t\in T:t$ is live under M_0.
5. Let the net N be live under M_0. The liveness of N satisfies monotonicity if $\forall\ M\geqslant M_0,N$ is live under M.

Definition 1.4. Let (N,M_0) be a net system and $N=(P,T;F)$. (N,M_0) is bounded iff $\exists\ k\in\mathcal{N},\forall\ M\in[M_0>,\forall\ p\in P:M(p)\leqslant k$.

Definition 1.5. Let $N=(P,T;F)$ be a net.

1. N is structurally bounded iff net system (N,M_0) is bounded for every M_0.
2. N is structurally live iff there exists an M_0 such that net system (N,M_0) is live.

Definition 1.6. Let $N=(P,T;F)$ be a net. $P'\subseteq P$, $T'={}^{\bullet}P'\cup P'^{\bullet}$, $F'=F\cap((P'\times T')\cup(T'\times P'))$. $N'=(P',T';F')$ is a subnet of N generated by P'.

1.2 Invariants

A net $N=(P,T;F)$ is pure iff $\forall\ p\in P,\forall\ t\in T:(p,t)\in F\wedge(t,p)\in F$ can not hold together. We consider pure net in this paper.

Definition 1.7. Let $N=(P,T;F)$ be a net and let $\mathcal{Z}$ be an integer.

1. A column vector $V:P\to\mathcal{Z}$ indexed by P is a P-vector.
2. A column vector $W:T\to\mathcal{Z}$ indexed by T is a T-vector.

3. A matrix $C: P\times T\rightarrow\mathscr{Z}$ indexed by P and T such that

$$c_{ij}=\begin{cases}-1 & \text{if } p\in {}^{\cdot}t\backslash t^{\cdot}\\ 1 & \text{if } p\in t^{\cdot}\backslash {}^{\cdot}t\\ 0 & \text{otherwise}\end{cases}$$

for all $p\in P$ and for all $t\in T$ is the incidence matrix of N.

We denote column vector where every component equals 0 by O.

Definition 1.8. Let $N=(P,T;F)$ be a net.

1. I is an S-invariant of N iff $I^T * C=O^T$.
2. $P_I\subseteq P$ is the support of I iff $P_I=\{p\in P\mid I(p)\neq 0\}$.

1.3 Deadlocks and traps

Definition 1.9. Let $N=(P,T;F)$ be a net.

1. A nonempty set $H\subseteq P$ is a deadlock iff ${}^{\cdot}H\subseteq H^{\cdot}$.
2. A nonempty set $H\subseteq P$ is a trap iff $H^{\cdot}\subseteq {}^{\cdot}H$.
3. Let H be a deadlock (trap). H is minimal iff there is no deadlock (trap) contained in H as a proper subset.
4. Let H be a deadlock (trap). H is maximal iff there is no deadlock (trap) that contains H as a proper superset.

A place $p\in P$ is marked by a marking M iff $M(p)>0$ and a nonempty set of places $H\subseteq P$ is marked by a marking M iff at least one element of H is marked. Obviously, if a deadlock lost all tokens it remains unmarked and if a trap gained at least one token it remains marked.

1.4 Some subclasses of P/T nets

Definition 1.10. Let $N=(P,T;F)$ be a net.

1. N is a Free-choice net iff $\forall p\in P: |p^{\cdot}|>1\Rightarrow {}^{\cdot}(p^{\cdot})=\{p\}$.
2. N is an Extended Free-choice net iff $\forall p_1,p_2\in P, p_1^{\cdot}\cap p_2^{\cdot}\neq\varphi\Rightarrow p_1^{\cdot}=p_2^{\cdot}$.
3. N is an Asymmetric choice net iff $\forall p_1,p_2\in P, p_1^{\cdot}\cap p_2^{\cdot}\neq\varphi\Rightarrow p_1^{\cdot}\subseteq p_2^{\cdot}\vee p_2^{\cdot}\subseteq p_1^{\cdot}$.

2 The Liveness and Monotonicity of AC Nets

2.1 The liveness of AC nets

Theorem 2.1. [9] An AC net system (N,M_0) is live if every (minimal) deadlock in N contains a marked trap.

By Theorem 2.1 we can judge the liveness for some AC net systems, but there exist some AC net systems which are live but have deadlocks containing unmarked or no traps. For example, the AC net shown in Fig. 1 is live even though the deadlock $\{p_1,p_2,p_3,p_4\}$ does not contain any trap.

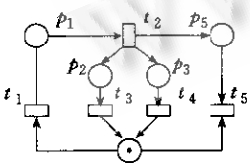

Fig. 1 A live AC net

From Ref. [14] we found a necessary and sufficient condition of liveness of AC nets.

Theorem 2.2. [14] An AC net system (N,M_0) is live iff for every minimal deadlock H in N, $\forall M\in[M_0>: M(H)\geq 1$.

We can decide whether an AC net system is live or not theoretically through Theorem 2.2. First, all reachable markings from M_0 and all minimal deadlocks have to be computed, then for every reachable marking M we have to judge whether every minimal deadlock is marked under M or not. For example, we will give the following outline of algorithm checking the liveness of AC net systems whose places number is n and is k-bounded.

Algorithm 2.1 (outline).

Input (N, M_0) and $N=(P,T;F)$ is an AC net, C is its incidence matrix.

Output Yes (N, M_0) is live.

No (N, M_0) is not live.

Step 1 Computing all reachable markings $[M_0>$.

If $M \in [M_0>$, $\exists\, t \in T$: $M[t>M'$ then $M'=M+C(-,t)$

$\{C(-,t)$ represents the t column of $C\}$

Step 2 Finding all minimal deadlocks. {The step can be referred to in [5].}

Step 3 For every reachable marking $M \in [M_0>$ and every minimal deadlock H,

if $M(H)=0$ then output "No",

else output "Yes".

We give the complexity analysis of Algorithm 2.1.

Theorem 2.3. Let $N=(P,T;F)$ be an AC net whose places number is n and is k-bounded. M_0 is its initial marking. The worst case time complexity of the Algorithm 2.1 is $O(n^2k^n)$.

Proof. (1) The places number of AC net is n and every place is k-bounded, then the worst case time complexity of getting $[M_0>$, i.e. Step 1, is $O(k^n)$.

(2) From Ref. [5], we know that the worst case time complexity of finding a deadlock is $O(|T|(|P|+|T|+|F|))$. But the number of minimal deadlocks is $|P|$ at most, so, the worst case time complexity of Step 2 is $O(|P||T|(|P|+|T|+|F|))$, i.e. $O(n^4)$.

(3) For every reachable marking M from M_0, checking whether a minimal deadlock is unmarked or not needs time $O(n)$. But, the number of reachable markings is k^n at most and the number of minimal deadlocks is n at most. So, the worst case time complexity of Step 3 is $O(n^2k^n)$. In many cases, $n^2k^n > n^4$, take the worst case time complexity of Step 3 as the algorithm complexity. □

Theorem 2.3 tells us that the time complexity of Algorithm 2.1 is exponential in the worst case. In the practical point of view it does not work. But, we know that an AC net system is live if and only if its every marked minimal deadlock cannot get unmarked through Theorem 2.2. So, we can try to find practical algorithms that can judge whether some AC net systems are live or not.

Definition 2.1. Let (N, M_0) be a net system, I be an S-invariant and $H \subseteq P$ be a deadlock of N. The deadlock H is controlled by the S-invariant I under M_0 iff $I^t * M_0 > 0 \wedge \forall\, p \in P \backslash H: I(p) \leqslant 0$.

Theorem 2.4. Let $\sum_0 = (N, M_0)$ be an AC net system. $\sum_0$ is live if every minimal deadlock in N is controlled by an S-invariant under M_0.

Proof. Let H be a minimal deadlock in N and I be an S-invariant. H is controlled by I under M_0, so $I^t * M_0 > 0 \wedge \forall\, p \in P \backslash H: I(p) \leqslant 0$. Since for $\forall\, M \in [M_0>: I^t * M_0 = I^t * M_0 > 0$, so only the places of H have positive entries in I. Hence, $M(D) > 1$. The AC net system $\sum_0$ is live according to Theorem 2.2. □

In order to check liveness for a given initial marking of a finite AC net through Theorem 2.4, we have to find all minimal deadlocks and S-invariants. Then, we check whether every minimal deadlock is controlled by an S-invariant under the initial marking. So, we can analyze the worst case time complexity of the algorithm based on Theorem 2.4 through the following Theorem 2.5.

Algorithm 2.2. (outline)

Input (N, M_0) and N is an AC net, C is its incidence matrix.

Output Yes (N, M_0) is live.

No liveness of (N, M_0) can not be judged.

Step 1 Finding all minimal deadlocks.

Step 2 Getting a basis of S-invariants for the solution of $X^T * C=0$ (X is a vector).

Step 3 For every minimal deadlock finding an S-invariant that controls it.

If the S-invariant can not be found then output "No",

else output "Yes".

Theorem 2.5. Let $N=(P,T;F)$ be an AC net and C its incidence matrix. The worst case time complexity of Algorithm 3.2 is $O(|P|^2(\max(|P|,|T|))^3)$.

Proof. (1) We know that the worst case time complexity of finding a minimal deadlock is $O(|T|(|P|+|T|+|F|))$ from Ref. [5]. The number of minimal deadlocks is $|P|$ at most, thus the worst case time complexity of getting all minimal deadlocks, i.e. Step 1, is $O(|P||T|(|P|+|T|+|F|))$.

(2) In order to get a basis of S-invariants, we need to solve the homogeneous linear equation system $X*C=0$. Through linear algebra, and we know its worst case time complexity is $O((\max(|P|,|T|))^3)$.

(3) We know that every S-invariant is a linear combination of the basis generated by Step 2. Judging whether a deadlock is controlled by an S-invariant or not needs at most $|P|$ time. So, for every minimal deadlock the worst case time complexity of checking whether it is controlled by an S-invariant is $O(|P|(\max(|P|,|T|))^3)$. The number of minimal deadlocks in a net is $|P|$ at most. Thus, the worst case time complexity of Step 3 is $O(|P|^2(\max(|P|,|T|))^3)$. Therefore, for Algorithm 2.2., its worst case time complexity is $O(|P|^2(\max(|P|,|T|))^3)$, i.e. polynomial time. □

Example 1. An AC net is shown in Fig. 1. $I=(1,1,1,1,-1)$ is its S-invariant. $I*M_0>0$, $H=\{p_1,p_2,p_3,p_4\}$ is a minimal deadlock and $I(p_5)=-1\leqslant 0$, H is controlled by I under M_0 and H is the unique minimal deadlock of the AC net. So we are sure the AC net system is live according to Theorem 2.4.

2.2 The liveness monotonicity of AC nets

The liveness of an AC net, generally, does not satisfy monotonicity. It is very important as we have a conjecture that there are some polynomial time algorithms for liveness if the liveness satisfies monotonicity, otherwise generally there are no polynomial time algorithms for liveness.

Theorem 2.6. [10~13] Let $N=(P,T;F)$ be an AC net. If $\sum_0=(N,M_0)$ is live, then $\forall M_i, M_i\geqslant M_0, \sum_i=(N,M_i)$ is live iff every nonempty minimal deadlock in N contains a trap marked under M_0. For bounded AC nets, we still can get a better result about liveness monotonicity.

Lemma 2.1. [10~13] Let $N=(P,T;F)$ be an AC net. H is its minimal deadlock iff H is strongly connected deadlock and for $\forall t\in H^{\cdot}: |{}^{\cdot}t\cap H|=1$.

Theorem 2.7. Let N be a structurally bounded AC net. Every minimal deadlock H of N contains a trap iff H is a trap.

Proof. "⇒" Since every minimal deadlock contains a trap, let M_0 be a marking that marks all these traps, so $\sum_0=(N,M_0)$ is live and bounded. Suppose there exists a minimal deadlock H which is not a trap. Let H' be its maximal trap, $H'\subset H$. So $\exists\, t\in {}^{\cdot}H'$ but $t\notin H'^{\cdot}$. Since $|{}^{\cdot}t\cap H'|=1$ according to Lemma 2.1, and $\sum_0$ is live. Occurring of t will increase tokens of H' and t can occur infinitely. Hence contradict the condition of boundedness. So, any minimal deadlock of N is a trap.

"⇐" Obvious. □

Theorem 2.8. Let $N=(P,T;F)$ be an AC net and $\sum_0=(N,M_0)$ be live and bounded. $\forall M_i, M_i\geqslant M_0, \sum_i=(N,M_i)$ is live iff every minimal deadlock of N is a marked trap under M_0.

Proof. "⇒" $\sum_0=(N,M_0)$ is live and its liveness satisfies monotonicity. By Theorem 2.6, every minimal deadlock of N contains a marked trap. Since $\sum_0$ is bounded, from the proof of Theorem 2.7 we know that every minimal deadlock of N is a trap. So every minimal deadlock of N is a marked trap.

"$\Leftarrow$" If every minimal deadlock of N is a marked trap under M_0, since $M_i \geqslant M_0$, every minimal deadlock of N is also a marked trap under M_i. So, $\sum_i = (N, M_i)$ is live. □

Example 2. An bounded and live AC net system is shown in Fig. 2. The net system will not be live if we add a token in p_5. Because $\{p_1, p_2, p_3, p_4\}$ is a minimal deadlock which is not a trap.

Theorem 2.8 simplifies judgement of Theorem 2.6 for liveness monotonicity of AC nets. We can directly get the following corollary through Theorem 2.8.

Corollary 2.1. Let $\sum_0 = (N, M_0)$ be an AC net system. If every minimal deadlock contains a marked trap, but there exists at least a minimal deadlock which is not a trap, then the AC net system $\sum_0$ is unbounded.

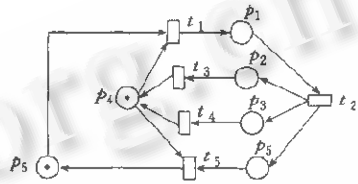

Fig. 2 A bounded and live AC net system

3 Conclusion

In this paper we have introduced a sufficient criterion for the liveness in AC net system by controlled deadlock. At the same time we also have got a necessary and sufficient condition for liveness monotonicity of AC nets. Although the conjecture about liveness monotonicity and practical way, i.e. polynomial time algorithms for the judgement of liveness, is still open, by analysis of liveness monotonicity of AC nets, we have known to a certain extent that we understand better about AC nets liveness. We may have chance in the future to search for the biggest subclass of AC nets satisfying liveness monotonicity and to discover and invent polynomial time algorithms for their liveness.

References:

[1] Reisig, W. Petri Nets, An Introduction. Berlin: Springer-Verlag, 1985.

[2] Esparza, J., Silva, M. A polynomial-time algorithm to decide liveness of bounded free-choice nets. Journal of Theoretical Computer Science, 1992, 102: 185~205.

[3] Barkaoui, K., Minoux, M. A polynomial-time graph algorithm to decide liveness of some classes of bounded Petri nets. LNCS, 1992, 616: 62~75.

[4] Lautenbach, K., Ridder, H. Liveness in bounded Petri nets which are covered by T-invariants. LNCS, 1994, 815: 358~375.

[5] Kemper, P., Bause, F. An efficient polynomial-time algorithm to decide liveness and boundedness of free-choice nets. LNCS, 1992, 616: 263~278.

[6] Desel, J. A proof of the rank theory for extended free choice nets. LNCS, 1992, 616: 134~153.

[7] Barkaoui, K., Couvreur, J.M., Duteilhet, C. On liveness in extended non self-controlling nets. LNCS, 1995, 935: 23~44.

[8] Minoux, M., Barkaoui, K. Polynomial time algorithms for proving or disproving commoner's structural property in Petri nets. In: Simpson, D. ed. Proceedings of the 9th International Conference on Application and Theory of Petri Nets. Venice Italy: IBM Deutschland, 1988.

[9] Murata, T. Petri nets, properties, analysis and applications. In: Proceedings of the IEEE, 1989, 77(4): 541~580.

[10] Lu, W., Zhen, Q. Research for the liveness of Petri net systems. Chinese Journal of Computers, 1999, 22(4): 1~4 (in Chinese).

[11] Zhen, Q., Lu, W. On liveness and safeness of asymmetric choice nets. Journal of Software, 2000, 11(5): 590~605 (in

Chinese).

[12] Zhen, Q. On liveness and boundedness of asymmetric choice nets [Ph.D. Thesis]. Institute of Mathematics, The Chinese Academy of Sciences, 1998.

[13] Zhen, Q., Lu, W. On liveness of asymmetric choice nets. Journal of Software, 1998,9(5):354～359 (in Chinese).

[14] Barkaoui, K., Pradat-Peyre, J-F. On liveness and controlled siphons in Petri nets. LNCS, 1996,1091:57～72.

附中文参考文献:

[10] 陆维明,甄强. Petri 网系统活性的研究. 计算机学报,1999,22(4):1～4.

[11] 甄强,陆维明. 论加权扩充自由选择网的活性和安全性. 软件学报,2000,11(5):590～605.

[13] 甄强,陆维明. 论非对称选择网的活性. 软件学报,1998,9(5):354～359.

非对称选择网活性的一个多项式时间判定

焦 莉, 陆维明

(中国科学院 数学与系统研究院数学研究所,北京 100080)

摘要: 活性判定是Petri 网中一直没有完全解决的问题. 针对非对称选择网的活性问题,利用结构分析理论,作了进一步的研究. 首先,讨论和分析了活性判定的一般方法,然后利用 S-不变,提出了非对称选择网活性判定的一个充分条件,并给出了相应的多项式算法. 同时,对有界的非对称选择网的活性单调性问题进行了深入的研究,得到了一个简单的充分必要条件.

关键词: 非对称选择网;活性;活性单调性;多项式时间

中图法分类号: TP301 **文献标识码:** A