

# 不同通信模型下的全光树环网波长分配算法\*

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## Wavelength Assignment Algorithms on Trees of Rings under Different Communication Models

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**Abstract:** This paper studies wavelength assignment algorithms on WDM all-optical trees of rings under different models: static, incremental and dynamic. It is shown that  $5L/2$  is the tight bound of the number of required wavelengths for static trees of rings with load  $L$ . This paper also proposes an  $O[\log_2(t+1)]$ -approximation and a  $\sum_{i=1}^h \max_{r \in R_i} \lceil \log |V(r)| \rceil + h$ -approximation algorithm for incremental and dynamic trees of rings respectively, where  $t$ ,  $h$  and  $R_i$  are the number of rings, the number of the layers of the underlying tree and the set of rings of layer  $i$  in the network respectively.

**Key words:** WDM; all-optical network; wavelength allocation; tree of rings; approximation ratio

**摘要:** 研究了波分复用全光树环网在不同通信模型下的波长分配算法及其最坏性能分析.对于静态模型,证明了  $5L/2$  是树环网所需波长数的紧界.对于动态模型,提出了一种近似比为  $\sum_{i=1}^h \max_{r \in R_i} \lceil \log |V(r)| \rceil + h$  的波长分配算法,其中  $h$  为树环网的基树的层数,  $R_i$  为树环网中处于第  $i$  层的环的集合,  $|V(r)|$  为环  $r$  上的节点数.对于增量模型,提出了一种近似度为  $O[\log_2(t+1)]$  的波长分配算法,其中  $t$  为树环网中的环数.

**关键词:** WDM;全光网;波长分配;树环;近似比

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## 1 Introduction

Optical network is emerging as a key technology in communication networks. In all-optical networks, the information reaches its final destination directly without being converted to electronic form in between once

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transmitted as light. WDM (wavelength division multiplexing) is a basic technology in all-optical networks. It partitions the available bandwidth on an optical fiber into some channels, each at a different wavelength. Each channel can carry a separate stream of data and any two streams of data must be assigned different wavelengths on a single optical fiber. Due to the natural congestion bound, it needs at least as many wavelengths as the *load* of an optical network, i.e. the maximum number of paths sharing a single link, to insure no blocking. In the following, we will always denote the *maximum load* and the *number* of the nodes of a considered network as  $L$  and  $N$  respectively. Utilizing the bandwidth efficiently is a critical aspect to improve the performance of a network.

There are three common models in the analysis of WDM all-optical networks: *static*, *incremental* and *dynamic*. Under the *static* model, all lightpath requests are given in advance, while under the *incremental* model, requests arrive as time goes by but are never terminated, and under the *dynamic* model, requests to set up lightpaths arrive over time and must be accommodated without rerouting the existing lightpaths, and lightpaths may be terminated over time as well.

### 1.1 Related Work

The wavelength allocation problem is known to be NP-hard for general WDM networks, even for some simple network topologies such as ring and tree<sup>[1]</sup>. Ring is a very popular topology and many remarkable results about wavelength allocation have been achieved on it. Under the static model, Gerstel *et al.* gave a lower bound  $2L-1$  in Ref.[2] and a tighter lower bound  $(2-2/(N+1))L$  can be found in Ref.[3]. Under the dynamic model, Gerstel *et al.*<sup>[4]</sup> presented an algorithm that uses at most  $L\lceil\log_2 N\rceil+L$  wavelengths and gave a general lower bound  $0.5L\lceil\log_2 N\rceil$ . Under the incremental model, Slusarek<sup>[5]</sup> proposed an optimal algorithm, which uses  $3L-2$  wavelengths. When wavelength conversion is allowed, Xu *et al.*<sup>[6]</sup> proved that the optimal utilization of the bandwidth can be achieved by placing a kind of converter of degree 4 at one node of a ring under the static model and that degree 4 is the lower bound to reach such performance if only one converter is allowed. Wan and Chen *et al.*<sup>[7]</sup> gave an optimal fixed conversion pattern for a static ring. Under the dynamic model,  $L\lceil\log_2 N\rceil+4L$  wavelengths are required if each of the nodes on the ring is equipped with a converter of degree 2<sup>[4]</sup>. Under the incremental model, the number of wavelengths needed is shown to be  $\max\{0, L-d\}+L$  for a conversion degree of  $d$  at each node<sup>[4]</sup>. Further achievements for the wavelength assignments in ring and star topologies can be seen in Refs.[13–15].

Tree is another common topology of networks. Under the static model, Kaklamani<sup>[8]</sup> and Erlebach<sup>[9]</sup> gave the best upper bound as  $5L/3$ . Kumar and Schwabe<sup>[10]</sup> gave a lower bound  $5L/4$ . Under the incremental model, Bartal and Leonardi<sup>[11]</sup> presented an  $O(\log_2 N)$ -approximation algorithm and proved that no deterministic algorithm for trees can have an approximation ratio better than  $\Omega(\log_2 N/\log_2 \log_2 N)$ . Under the dynamic model, an algorithm that requires no more than  $(2L-1)\lceil\log_2 N\rceil$  wavelengths is proposed in Ref.[4].

For trees of rings, Deng *et al.*<sup>[12]</sup> showed that  $5L/2$  is the upper bound under the static model. Combining wavelength allocation with routing, Bartal *et al.* proved that there exists an on-line algorithm for trees of rings which is  $O(\log_2 N)$ -approximation<sup>[11]</sup>. Star-ring is a kind of topology in which some sub-rings are connected by a backbone ring. It is a special form of trees of rings.

### 1.2 Summary of Results

This paper studies wavelength allocations on trees of rings in the worst cases under all the three common models. No wavelength conversion is available, and for convenience, we assume that each ring in the network is associated with the *counterclockwise direction*. Under the static model, we present a sequence of requests with maximum load  $L$ , which requires at least  $5L/2$  wavelengths for any algorithm. This shows that the  $5/2$ -approximation given by Deng *et al.*<sup>[12]</sup> is optimal. Under the incremental and the dynamic model, we classify

the rings in a tree of rings into some layers and present two approximation algorithms based on the classification, one for each model. The one for the incremental model is with an approximation ratio  $O[\log_2(t+1)]$ , where  $t$  is the number of rings in the network. This improves the  $O(\log_2 N)$ -approximation algorithm presented by Batal and Leonardi<sup>[11]</sup>. The one for the dynamic model is with an approximation ratio  $\sum_{i=1}^h \max_{r \in R_i} \lceil \log |V(r)| \rceil + h$ , where  $R_i$  is the set of rings of layer  $i$ ,  $V(r)$  is the set of nodes on ring  $r$ , and  $h$  is the number of layers of the underlying tree of the network.

### 2 Preliminary

An optical network can be represented as a graph  $G=(V(G),E(G))$ . Under many models of optical routing, a set  $P$  of dipaths in  $G$  is given and different dipaths sharing a link must be assigned different wavelengths on the link. In the following, we denote  $W_G(L)$  as the number of the required wavelengths to be assigned to lightpaths in a network with topology  $G$  without blocking.

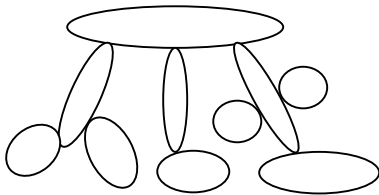


Fig.1 An example of trees of rings

**Definition 1.** A tree of rings corresponds to a tree, which is called its *underlying tree*. Each node on the underlying tree corresponds to a ring on the tree of rings, while each edge corresponds to the common node shared by two corresponding rings. See Fig.1.

In a tree of rings, the number of rings equals to the number of nodes of its underlying tree, and the common nodes shared by two rings are called *joint nodes*. A lightpath in a tree of rings may be divided into several segments on different rings by the joint nodes it traverses. For example, lightpath  $p_2$  is divided into three segments in  $Ring_1$ ,  $Ring_2$  and  $Ring_3$  respectively in Fig.2. There are two cases that two lightpaths traversing the same ring  $R$  may overlap each other in another ring: (1) their corresponding segments in  $R$  are *adjacent*, i.e., the source node of one lightpath is the same as the destination node of the other,  $p_1$  and  $p_2$  in  $Ring_2$  in Fig.2 for example; (2) their corresponding segments in  $R$  overlap each other,  $p_2$  and  $p_3$  in  $Ring_2$  in Fig.2 for example. The segment of a lightpath in a specific ring may be regarded as an independent lightpath within the ring by algorithms in the following. The technique of dividing lightpaths into segments will be employed in the analyses of wavelength allocations under the dynamic and incremental models. The character in case (1) stated above is crucial in the analyses of the following algorithms.

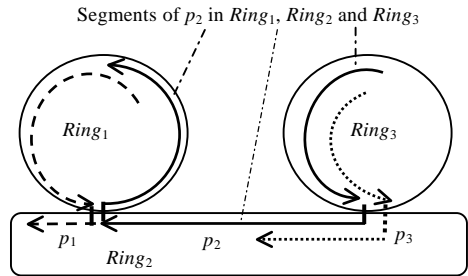


Fig.2 Divisions of lightpaths by joint nodes

### 3 The Tight Bound for Trees of Rings under the Static Model

For trees of rings under the static model, Deng *et al.*<sup>[12]</sup> proposed a  $5/2$ -approximation algorithm. In this section, we will show that  $5L/2$  is also the tight bound in the worst case by presenting a set of requests which need at least  $5L/2$  wavelengths for any algorithm.

**Theorem 1.** For a general tree of rings,  $W_{tree-rings}(L)=5L/2$  is the tight bound in the worst case under the static model.

*Proof.* It follows that  $W_{tree-rings}(L) \leq 5L/2$  from Ref.[12]. In the following, we will give a set of requests that need at least  $5L/2$  wavelengths for any algorithm. Therefore,  $W_{tree-rings}(L) \geq 5L/2$  in the worst case. And Theorem 1 concludes.

Given a tree of rings, five sub-rings labeled  $R_0, R_1, R_2, R_3$  and  $R_4$  are connected by a backbone ring  $R$  counterclockwise. See Fig.3(a). Let  $P_i(0 \leq i \leq 4)$  be a set of  $L/2$  identical lightpaths, whose source node is in  $R_i$ , and whose destination node is in  $R_{(i+2) \bmod 5}$ . All lightpaths in  $P_i$  overlap all lightpaths in  $P_{(i+2) \bmod 5}$ . See Fig.3(b). Then it can be seen that all lightpaths in  $P_i$  overlap all lightpaths in  $P_j, j \neq i$ . So all the lightpaths in  $\cup_{i=0}^4 P_i$  overlap each other and thus cannot be assigned the same wavelength with each other. Therefore at least  $5L/2$  wavelengths are required for this set of requests for any wavelength allocation algorithm, while the load of the tree of rings is  $L$ . Theorem 1 holds.

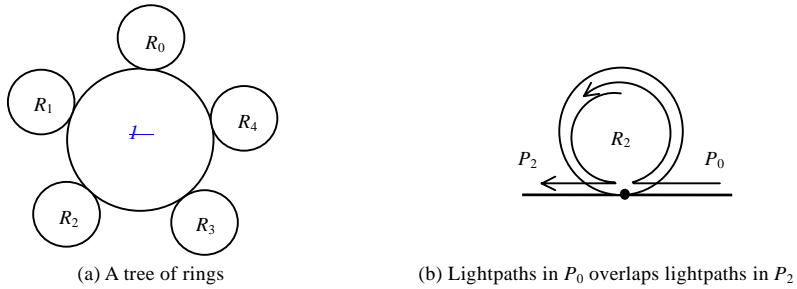


Fig.3

#### 4 Dynamic Wavelength Allocation on Trees of Rings

In this section, we study dynamic wavelength allocation on trees of rings, where the existing lightpaths cannot be rerouted and no blocking of lightpath is allowed as long as the maximum load does not exceed  $L$ . In this section, we will first give a special property (Lemma 1) of the efficient dynamic wavelength allocation algorithm DWLA<sup>[4]</sup> for ring networks given by Gerstel *et al.* and then propose a dynamic wavelength allocation algorithm for trees of rings based on DWLA and Lemma 1. In DWLA, the available wavelengths are sorted into several disjoint pools. The following Lemma 1 points out a feature of DWLA, which is crucial to our algorithm for trees of rings.

**Lemma 1.** Due to DWLA, two adjacent lightpaths in a ring network will never be assigned the same wavelength.

*Proof.* Given two adjacent lightpaths  $P_1$  and  $P_2$  in the ring:

*Case 1:* At least one of them crosses link 0. Let  $P_1$  crosses link 0. If  $P_2$  crosses link 0 too,  $P_1$  and  $P_2$  would be assigned two different wavelengths in  $Pool(\lceil \log_2 N \rceil)$  by DWLA. Otherwise, a wavelength in  $Pool(\lceil \log_2 N \rceil)$  would be assigned to  $P_1$ , while a wavelength in another pool to  $P_2$ .

*Case 2:* None of them crosses link 0 (See Fig.4). Let, by contradiction,  $P_1$  and  $P_2$  be assigned the same wavelength in some  $Pool(m)(0 \leq m < \lceil \log_2 N \rceil)$  by DWLA. In this case,  $P_1$  crosses link  $a2^m$ , while  $P_2$  crosses another

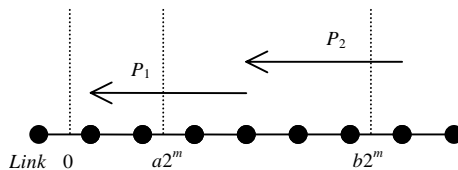


Fig.4 The on-hub segments of two lightpaths

link  $b2^m$  for some two different odds  $a$  and  $b$ . Assume that  $a < b$  and  $b - a = 2c$  ( $c > 0$ ). We have  $b2^m - a2^m = c2^{m+1}$ . So there exists some link  $k2^{m+1}$ , where  $a2^m < k2^{m+1} < b2^m$ . Since  $P_1$  and  $P_2$  are adjacent to each other, either  $P_1$  or  $P_2$  crosses link  $k2^{m+1}$ . Due to DWLA, at least one of them should have been assigned a wavelength in a higher pool than  $Pool(m)$ . It is a contradiction. Lemma 1 holds.

Recall that two lightpaths traversing the same ring  $R$  may overlap in another ring only if their segments in  $R$  are adjacent to or overlapping each other. Lemma 1 implies that if we use DWLA to assign wavelengths to the lightpaths traversing ring  $R$  just based on their segments in  $R$ , there will be no conflict among all these lightpaths whether on  $R$  or on other rings. Before introducing our algorithm, let us explain our strategy of classifying the rings in a given tree of rings.

**Definition 2.** Given a non-trivial tree with  $t$  nodes, we're always able to pick out a node, the removal of which divides the tree into at least two sub-trees, each of which has no more than  $t/2$  nodes. Such a node is called a *cut node* of the tree.

From Definition 2, we can classify all the nodes of a tree into some layers as follows: given a non-trivial tree, pick out a cut node of the tree as the node of *layer 1*, remove it and leave several sub-trees to the next step. Then pick out a cut node in each sub-tree as the nodes of *layer 2*, and so on, until all the sub-trees become empty. The rings in a tree of rings can also be sorted into some layers in the same way based on its underlying tree. In the following we denote *the number of the layers* as  $h$ . It is obvious that  $h \leq \lceil \log_2(N+1) \rceil$ .

The above idea is inspired by the argument of wavelength allocation for dynamic tree networks in Ref.[4]. Our dynamic wavelength allocation algorithm is implied in the following Theorem 2. It employs the following Conclusion 1.

**Conclusion 1.** A path traversing through two nodes of layer  $k$  ( $k > 1$ ) in a tree contains a node of layer lower than  $k$ . A path traversing through two rings of layer  $k$  ( $k > 1$ ) in a tree of rings traverses through a ring of layer lower than  $k$ .

**Theorem 2.** Let  $TR$  be a tree of rings. Then  $W_{TR}(L) \leq (\sum_{i=1}^h \max_{r \in R_i} \lceil \log |V(r)| \rceil + h)L$ , where  $R_i$  is the set of rings of layer  $i$ ,  $|V(r)|$  is the number of nodes on ring  $r$  and  $h$  is the number of the layers of  $TR$ .

*Proof.* Let there be  $(\sum_{i=1}^h \max_{r \in R_i} \lceil \log |V(r)| \rceil + h)L$  wavelengths available in  $TR$ . We classify all the rings in  $TR$  into  $h$  layers as the above, and divide the available wavelengths into  $h$  disjoint pools, where  $Pool(i)$  has  $(\max_{r \in R_i} \lceil \log |V(r)| \rceil + 1)L$  wavelengths and is for the wavelength allocation of lightpaths on rings of layer  $i$ .

Given an incoming lightpath  $p$ , let  $r_1$  be the ring of the lowest layer, say layer  $l_1$ , which  $p$  traverses. Note that such an  $r_1$  is unique for every lightpath in the network due to our layer-classifying strategy. Let the nodes in  $r_1$  be labeled from  $j_1$  to  $j_{v_{r_1}}$ . We consider the segment of  $p$  in  $r_1$ , say  $p_1$ , as a separate lightpath, and use DWLA to select an available wavelength in  $r_1$  out of  $Pool(l_1)$  for  $p_1$  and further more for  $p$ . Gerstel *et al.*<sup>[7]</sup> concluded that

$W_{ring}^{DWLA}(N, L) \leq L \lceil \log_2 N \rceil + L$ , where  $N$  is the number of nodes in the ring. So there is an available wavelength for

$p_1$  from the view of ring  $r_1$  because there are  $(\max_{r \in R_i} \lceil \log |V(r)| \rceil + 1)L$  wavelengths for layer  $i$  ( $1 \leq i \leq h$ ).

For another lightpath  $q$ , we will show that  $q$  is assigned a different wavelength from  $p$  if  $q$  overlaps  $p$ . Denote the ring of the lowest layer which  $q$  traverses as  $r_2$  and of layer  $l_2$ . We denote the segment of  $q$  in  $r_2$  as  $q_1$ . If  $l_1 \neq l_2$ , then  $Pool(l_1) \neq Pool(l_2)$ . So  $q$  is assigned a wavelength different from  $p$  in another pool. If  $l_1 = l_2$  and  $r_1 \neq r_2$ , it can be

concluded that  $p$  or (and)  $q$  traverses a ring of lower layer than  $l_1=l_2$  from Conclusion 1 since  $p$  and  $q$  overlap each other. It contradicts to the assumption that  $l_1=l_2$  is the lowest layer that  $p$  and  $q$  traverse. If  $l_1=l_2$  and  $r_1=r_2$ ,  $p_1$  and  $q_1$  are adjacent to or overlapping each other in  $r_1=r_2$  since  $p$  and  $q$  overlap in some ring(s). So  $q$  is assigned a wavelength different from  $p$  from Lemma 1 and DWLA. Theorem 2 holds.

## 5 Wavelength Allocation under the Incremental Model

The incremental model is suitable for networks with growing demands and with almost no requirements for removing lightpaths that are already in use. In this section, we present an  $O[\log_2(t+1)]$ -approximation algorithm for incremental trees of rings based on a modification of the algorithm COLOR for rings proposed by Sluarek<sup>[5]</sup>, where  $t$  is the number of rings in the network. COLOR uses at most  $3L-2$  wavelengths for incremental ring networks. It divides  $3L-2$  wavelengths into  $L$  pools:  $Pool(0), Pool(1), \dots, Pool(L-1)$ , where  $Pool(0)$  contains one wavelength and each of the others contains three. An incoming lightpath is sorted into some shelf by the load it experiences and will be assigned a wavelength in the corresponding pool to it. The set of lightpaths with a wavelength in  $Pool(i)$  is denoted as  $Shelf(i)$ ,  $i = 0, 1, \dots, L-1$ . For a lightpath  $p$ , denote  $L(p/S) = \max_{e \in p} L(e/S)$ , where  $L(e/S)$  is the number of lightpaths in set  $S$  (not including  $p$  if  $p \in S$ ) that traverse link  $e$ .

In order to deal with wavelength allocation on trees of rings, a stronger constraint that no pair of *adjacent lightpaths* can be assigned the same wavelength is added to the incremental model of ring networks. The pseudo-code of the modified COLOR, COLOR\_AC, is shown in Fig.5. It is just the same as COLOR except that it has an additional constraint and there are three wavelengths in  $Pool(0)$ .

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### COLOR\_AC

*Input:* A sequence of requests to add lightpaths  $p_0, p_1, \dots$ , one at a time, where the load of the ring is at most  $L$ .

*Data Structure:*

1. A collection  $\{Shelf(i)\}_{i=0}^{L-1}$ , where  $Shelf(i)$  is a set of lightpaths.
2. A collection of pools  $\{Pool(i)\}_{i=0}^{L-1}$ , where each pool contains 3 different consecutive wavelengths. The lightpaths in  $Shelf(i)$  will be accommodated by  $Pool(i)$ .

*Additional Constraint:* No pair of adjacent lightpaths can be assigned the same wavelengths even if they're in the same shelf.

*Initialization:* For each  $i \geq 0$ , set  $Shelf(i) = \Phi$ .

*Processing a Request:* Upon arrival of a new lightpath request  $p$ :

1. Set  $i=0$ ;
  2. While  $L(p/Shelf(0) \cup \dots \cup Shelf(i)) > i$  do set  $i=i+1$ ;
  3. Set  $Shelf(i) = Shelf(i) \cup \{p\}$ ;
  4. Accommodate  $p$  using wavelengths in  $Pool(i)$  without violating the additional constraint.
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Fig.5 Description of COLOR\_AC

We will first show that COLOR\_AC requires no more than  $3L$  wavelengths for incremental ring networks without blocking lightpaths, and then give the incremental algorithm for trees of rings based on it. We list the lightpath requests as  $p_1, p_2, \dots, p_k$ , according to the order of their arrival up to some given time  $T$ . Denote  $F_i = \{p_1, \dots, p_{i-1}\}$  as the set of lightpaths which arrive before  $p_i$  and  $T_i = \bigcup_{j=0}^i Shelf(j)$  as the set of lightpaths in shelves 0 to  $i$  at time  $T$ . Then  $F_x \cap T_y$  denotes the set of lightpaths in *Shelves* 0 to  $y$  at the time when  $p_x$  arrives.

The following three lemmas conclude for COLOR. Lemma 2 states that when a lightpath is put into  $Shelf(i)$ , the maximum load it experiences in shelves 0, ...,  $i-1$  does not occur in segments where it overlaps other lightpaths in  $Shelf(i)$  for  $i > 0$ . Lemma 3 states that the path of a pair of lightpaths in  $Shelf(i)$  cannot fully contain each other for

$i > 0$ .

**Lemma 2.**<sup>[5]</sup> If  $p_x \cap p_y \neq \emptyset$  and  $p_x, p_y \in Shelf(i)$  for some  $x < y$  and  $i > 0$ , then  $L(p_x \cap p_y / F_y \cap T_{i-1}) \leq i - 1$ .

**Lemma 3.**<sup>[5]</sup> If  $p_x, p_y \in Shelf(i)$  for some  $x < y$  and  $i > 0$ , then neither  $p_x \subseteq p_y$  nor  $p_y \subseteq p_x$ .

**Lemma 4.**<sup>[5]</sup> The lightpaths belonging to  $Pool(0)$  do not overlap. The maximum number of lightpaths that belong to  $Shelf(i)$  and overlap on a given link is 2 for  $i > 0$ .

Under the model with additional constraint, we call two lightpaths *conflict* with each other if they are adjacent or overlap. The following Lemma 5 shows that if a lightpath  $p$  in  $Shelf(i)$  is adjacent to another lightpath  $q$  in  $Shelf(i)$  at one of its end node  $v$ , then  $p$  doesn't conflict with any lightpath in  $Shelf(i)$  except  $q$  at  $v$ .

**Lemma 5.** Given  $p_x, p_y \in Shelf(i)$  for  $i > 0$ . If  $p_x$  is adjacent to  $p_y$  and their common node is  $v$ , then  $v \notin p_z$  for any  $p_z \in Shelf(i)$ , where  $p_z \neq p_x$  or  $p_y$ .

*Proof.* If  $v \in p_z$  by contradiction. Since  $p_x$  is adjacent to  $p_y$  at node  $v$ ,  $p_z$  overlaps  $p_x$  or (and)  $p_y$ . Without loss of generality, we assume that  $p_z$  overlaps  $p_y$ . If  $p_z$  is also adjacent to  $p_x$  at node  $v$ , then  $p_z \subseteq p_y$  or  $p_y \subseteq p_z$ . It's a contradiction to Lemma 3. Otherwise,  $p_z$  overlaps  $p_x$  as well. Let the lightpaths be denoted by their end-nodes as  $p_x = (u, \dots, v)$ ,  $p_y = (v, \dots, w)$ ,  $p_z = (w_1, \dots, w_2)$ . Label all the nodes of the ring as 1, 2, ...,  $N$  sequentially, starting from node  $u$  through node  $v$  to node  $w$ . We have  $p_x = (s_1, \dots, a)$ ,  $p_y = (a, \dots, e_1)$ ,  $p_z = (s_2, \dots, e_2)$ , where  $a, s_1, s_2, e_1$  and  $e_2$  are the labels of node  $v, u, w_1, w$  and  $w_2$  respectively. From Lemma 3, none of  $p_x, p_y$  and  $p_z$  is contained in the other. So  $s_1 < s_2 < a$  and  $a < e_2 < e_1$ . It follows that  $L(p_x \cap p_z / F_{\max(x,z)} \cap T_{i-1}) \leq i - 1$  and  $L(p_y \cap p_z / F_{\max(y,z)} \cap T_{i-1}) \leq i - 1$  from Lemma 2. Since  $p_z = (p_x \cap p_z) \cup (p_y \cap p_z)$  and the load only grows as more lightpaths are added,  $L(p_z / F_z \cap T_{i-1}) \leq i - 1$  and  $p_z$  would have been placed in a lower shelf than  $Shelf(i)$  by COLOR-AC. It's a contradiction. Lemma 5 concludes.

From Lemmas 3, 4 and 5, a lightpath in  $Shelf(i)$  ( $i > 0$ ) may conflict with at most two other lightpaths in  $Shelf(i)$ , one at each side. Therefore, given an incoming lightpath in  $Shelf(i)$  for  $i > 0$ , the algorithm is always able to find a wavelength for it, which is different from its conflicting lightpaths in  $Shelf(i)$ , out of the three wavelengths in  $Pool(i)$ . It can also be seen that three wavelengths are necessary and sufficient for  $Pool(0)$  under the additional constraint in COLOR-AC, while it needs only one wavelength in COLOR. From the above, the following Theorem 3 concludes.

**Theorem 3.** When no pair of adjacent lightpaths with the same wavelength is allowed, COLOR-AC uses at most  $3L$  wavelengths for an incremental ring.

Using algorithm COLOR\_AC as well as COLOR to deal with the wavelength allocation problem on trees of rings, we have the following Theorem 4.

**Theorem 4.** Under the incremental model,  $W_{tree-rings}(L) \leq 3hL - 2$ , where  $h$  is the number of the layers of the underlying tree of the network.

*Proof.* The argument is similar to Theorem 2. All the rings in the network are classified into  $h$  layers. And  $3hL - 2$  wavelengths are divided into  $h$  disjoint pools:  $Pool(1), Pool(2), \dots, Pool(h)$ , where  $Pool(h)$  contains  $3L - 2$  wavelengths and each of the others contains  $3L$  wavelengths.

For an incoming lightpath  $p$ , let  $r$  be the ring of the lowest layer it traverses and be of layer  $k$  ( $1 \leq k \leq h$ ).  $p$  will be assigned a wavelength in  $Pool(k)$  according to its segment in  $r$ . Such a segment will be considered as a lightpath within  $r$  by the algorithm. If  $k < h$ , we view the wavelength allocation for  $p$  as a stronger problem that no pair of adjacent lightpaths in  $r$  can be assigned the same wavelength in order to avoid conflict. COLOR\_AC can solve this problem using no more than  $3L$  wavelengths within a ring from Theorem 3. Moreover, similar to the proof of Theorem 2, it can be seen that  $p$  doesn't overlap other lightpaths with wavelengths in  $Pool(k)$  but don't traverse  $r$  due to our classification strategy. So it can be assigned a wavelength independent from these lightpaths. Note that COLOR can be used especially for the lightpaths within a ring of layer  $h$  and only  $3L - 2$  wavelengths are required.

Hence,  $3hL-2$  wavelengths can accommodate all the lightpaths in a tree of rings whose expected maximum load is  $L$ .

Recall that  $h \leq \lceil \log_2(t+1) \rceil$ . The algorithm in Theorem 4 is  $O[\log_2(t+1)]$ -approximation, where  $t$  is the number of nodes in the underlying tree of the network. This algorithm improves the  $O(\log_2 N)$ -approximation one proposed by Batal and Leonardi<sup>[11]</sup>, where  $N$  is the total number of nodes in the network.

## 6 Conclusion

This paper analyzes the wavelength allocation problem on WDM all-optical trees of rings under three different models. We show that the bound  $5L/2$  is tight under the static model. We also propose an approximation algorithm for dynamic model. What's more, we improve the approximation ratio from  $O(\log_2 N)$  to  $O[\log_2(t+1)]$  under the incremental model, where  $N$  and  $t$  are the number of nodes in the network and the number of rings in the network respectively.

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